FDTD Signal Extrapolation Using the Forward-Backward Autoregressive (AR) Model

Vikram Jandhyala, Eric Michielssen, and Raj Mittra, *Fellow, IEEE*

***Abstract*—In this paper, the forward-backward autoregressive model is employed to extrapolate time domain signatures generated by the Finite Difference Time Domain algorithm with a view to speeding up the process of obtaining the frequency responses of EM systems. It is shown that the present method requires considerably lower-order predictors than those needed in other AR models, e.g., those based on Yule-Walker equations, and is therefore computationally efficient.**

I. Introduction

The FINITE DIFFERENCE Time Domain (FDTD) algorithm is a useful tool for electromagnetic simulation of complex structures. It generates a time signature that, upon Fourier transformation, yields the frequency response of the system being simulated. Frequently, the FDTD program has to be run for a large number of time steps to achieve good accuracy in the frequency domain results, and this can be computationally expensive. One approach to circumventing this problem is to extrapolate the time signature to late times after truncating the time iteration to a reasonable number of initial time steps. A number of methods such as autoregression (AR) [1], generalized-pencil-of-functions [2], and Prony's method [3] have been employed for extrapolation of FDTD signatures. In this paper, we use the forward-backward autoregressive model [4] for signal extrapolation and we show that it has definite advantages over the autocorrelation- and covariance-based AR methods.

II. AR Modeling

The autoregressive model is a prediction method that has found wide applications in several fields. Unlike many interpolation and extrapolation techniques that are deterministic curve fits, the AR method is stochastic and takes advantage of the statistics of the data.

If a discrete time series $x(1), x(2), \dots, x(p)$ is to be modeled using AR, then the following model is assumed:

$$x(n) = -a_1x(n-1) - a_2x(n-2) - \dots - a_px(n-p) + q(n) \quad (1)$$

The constants $a_1, \dots, a_p$ are the AR parameters that need to be determined from the signal $x(n)$, and $q(n)$ is a white noise process whose variance also has to be found to carry out the extrapolation of the signal.

Manuscript received January 26, 1994. This work was supported in part by the Joint Services Electronics Program under Grant N00014-90-J-1270.

The authors are with the Electromagnetic Communication Laboratory, Electrical and Computer Engineering Department, University of Illinois, Urbana, IL 61801 USA.

IEEE Log Number 9402609.

There are a whole host of approaches to evaluating these AR parameters. The more straightforward ones are based upon the use of the Yule-Walker equations [4], which involve the use of autocorrelation or covariance estimation. These methods suffer from certain difficulties, however, notable among which is that the AR parameters they yield are often not very accurate. This is because the ensemble calculation required to find the autocorrelation or covariance (expectation of a random signal of the form $E[\mathbf{XX}^T]$) is substituted in these methods by using the law of large numbers and by approximating the autocorrelation or covariance with inexact functions of the known time signal. These approximations impact the extrapolation results in two ways. The use of a low-order AR model in these methods causes the extrapolated signal to attenuate very rapidly. To compensate for this, a very high-order AR model is required that, in turn, can cause divergence problems in some cases because of the general statistical instabilities introduced by the large order. These two effects have been observed in the extrapolation of FDTD signals using the covariance method.

The forward-backward AR method [4], [5] avoids these problems by working directly with the data, rather than calculating the autocorrelation or covariance functions of the data.

III. Numerical Results

The inset of Fig. 1 shows the structure that was used as a test example for FDTD computation and extrapolation. The structure consists of two identical regions, one below the other. The upper region is free space and a sinusoidal point source of the form $V_s = u(t)\sin wt$, where $u(t)$ is the unit step function, is placed at the center of the region. The operating frequency is 1 GHz. The lower half has a relative dielectric constant of 50 and a conductivity of 0.001 S. The size of the total structure is $75 \times 75 \times 30$ cm^3 and it is uniformly divided into 50000 cubical cells. The lower region contains a conducting plate of dimensions $10 \times 10 \times 1$ (in cells). It is located as shown in the figure. The time step is 0.025 ns.

Fig. 1 shows the computed voltage signature at the observation point A and the extrapolated signal calculated using forward-backward AR. Fig. 2 compares the normalized log spectra of the actual 5000-time-step waveform, with the extrapolated waveform generated by using the forward- backward AR, and also with the AR result based upon the solution of the Yule-Walker equations. For both the time signatures, the Yule-Walker AR gave very poor results in the time domain when compared with the forward-backward AR. These signals

1051-8702/94$04.00 © 1994 IEEE

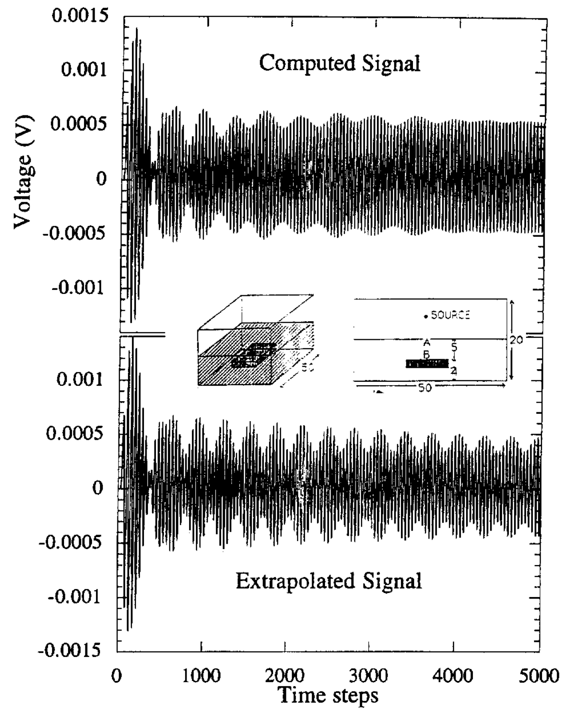

Fig. 1. The inset shows the structure being analyzed and the observation points A and B. The upper signal is the computed voltage signature at point A and the lower signal is the extrapolated signal.

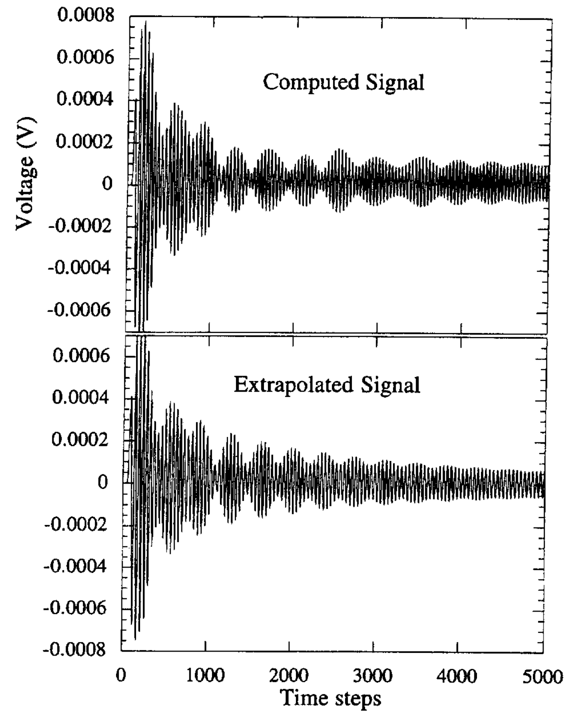

Fig. 3. The upper signal is the computed voltage signature at point B, and the extrapolated signal is shown below.

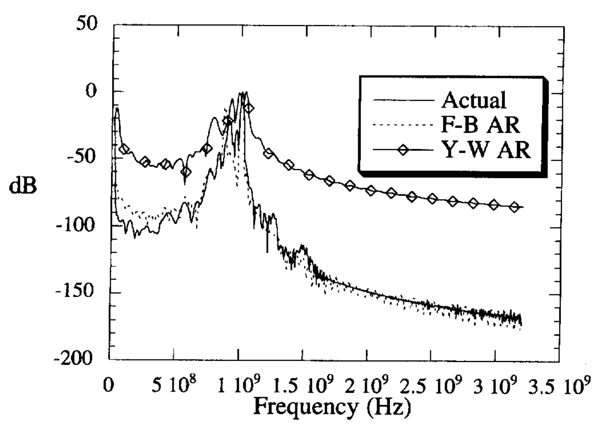

Fig. 2. Normalized log spectra for signals at point A. The actual computed spectrum is compared with those obtained from Yule-Walker AR and Forward-Backward AR.

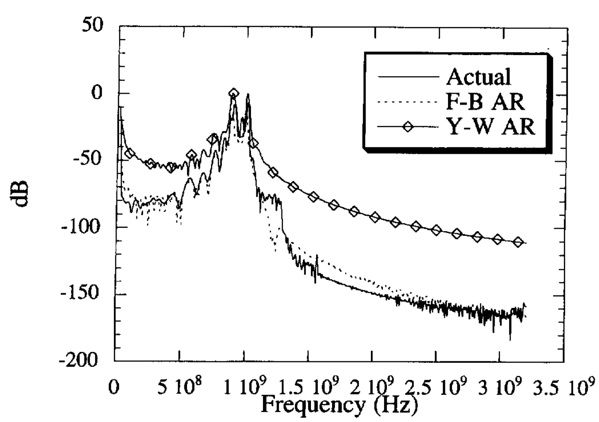

Fig. 4. Normalized log spectra for signals at point B, showing the computed spectrum, and the spectra from Yule-Walker AR and Forward-Backward AR.

attenuated very quickly and approached zero after 2000 time steps. Both the AR methods are trained on the same data set, comprising the data points from 201 to 800 time steps. The forward-backward AR model uses a 5th-order predictor, while the Yule-Walker AR model uses a 50th-order predictor.

Fig. 3 shows the computed voltage signature at the observation point B and the extrapolated signal. The normalized log spectra are shown in Fig 4. The training set for both the AR methods is comprised of the data points ranging from 201 to 800 time steps. The forward-backward AR model uses a 5th-order predictor, while a 100th-order predictor is used for the Yule-Walker AR model.

In both cases, appropriately decimated and low-pass filtered versions of the input signals were used for the Yule-Walker AR.

IV. Conclusion

The numerical results show that, for the signals considered, extrapolation with the forward-backward AR yields superior results to that obtained by using the Yule-Walker AR. This is because approximations of autocorrelation or covariance functions are not needed in the forward-backward AR and, hence, the accuracy of the AR parameters is better.

Marple [5] has described a recursive algorithm for obtaining the AR parameters using the forward-backward AR and has shown that it has the same computational complexity as the Levinson-Durbin recursive algorithm used for the Yule-Walker AR. This algorithm, and the observation that the forward-backward method requires a much lower-order predictor than is needed in the Yule-Walker approach, together imply that the forward-backward AR method is faster than the Yule-Walker method. This is what has been observed in our simulations, and although one cannot guarantee this behavior for all cases, it is expected that the same will be true for a majority of situations.

Yet another advantage of the forward-backward AR method is that the chances for spectral line splitting and false peak appearance that often plague the covariance-based AR [6], are reduced.

In conclusion, then, the forward-backward AR algorithm appears to provide a very good approach to FDTD signal extrapolation, both in terms of accuracy and speed.

REFERENCES

[1] J. Chen, C. Wu, K. Wu, and J. Litva, "Combining an autoregressive model with the FDTD algorithm for improved computational efficiency," *IEEE MTT-S Dig.*, pp. 749–752, 1993.
[2] Y. Hua and T. K. Sarkar, "Generalized pencil-of-function method for extracting poles of an EM system from its transient response," *IEEE Trans. Antenna Propagat.*, vol. 37, pp. 229–234, Feb. 1989.
[3] W. L. Ko and R. Mittra, "A combination of FD-TD and prony's method for analyzing microwave integrated circuits," *IEEE Trans. Microwave Theory Tech.*, vol. 39, pp. 2176–2181, Dec. 1991.
[4] S. M. Kay and S. L. Marple, "Spectrum analysis—a modern perspective," *Proc. IEEE*, vol. 69, pp. 1380–1419, Nov. 1981.
[5] S. L. Marple, "A new autoregressive spectrum analysis algorithm," *IEEE Trans. Acoustics, Speech, Sig. Proc.*, vol. ASSP-28, pp. 441–454, Aug. 1980.
[6] D. N. Swingler, "A modified Burg algorithm for maximum entropy spectral analysis," *Proc. IEEE*, vol. 67, pp. 1368–1369, Sept. 1979.