Topics:
1. Band-to-band optical transitions
2. Absorption spectrum and mechanisms
3. Spontaneous emission
4. LED principle and efficiency
5. Frequency response and modulation bandwidth

Reading: Liu, Sec. 13.2, 13.4-5, 13.7
Ref: Bhattacharya, Sec. 3.1-3.4, 5.4-5.5, 5.8
Griffith, "Quantum Mechanics!"

x. Energy and Momentum Conservation

Electron wavevector $|\vec{k}| \sim \frac{2\pi}{a}$

$\alpha$: lattice constant, $< 1\text{ nm}$.

Photon wavevector $|\vec{k}_{\text{photon}}| = \frac{2\pi}{\lambda} \ll |\vec{k}|$

$\hbar \vec{k}$: Crystal momentum of $e^-$ in a band state.

of wavevector $\vec{k}$.

$\hbar \vec{k}_{\text{photon}}$: Photon momentum

Hole energy: $E_1 = E_V - \frac{\hbar^2 k^2}{2m_n^*}$

Electron energy: $E_2 = E_C + \frac{\hbar^2 k^2}{2m_e^*}$

Exercise: Incoming photon energy $E_{ph} = \hbar \nu$

\[
E_1 = E_V - \frac{m_n^*}{m_e^*} (\hbar \nu - E_g)
\]

\[
E_2 = E_C + \frac{m_e^*}{m_n^*} (\hbar \nu - E_g)
\]

$M^* = \frac{m_e^* m_n^*}{m_e^* + m_n^*}$: Reduced effective mass

~ Indirect transition:

Involving absorption or emission of a phonon of energy $E_p$ and a wavevector $\vec{k}_p$.

$E_2 - E_1 = \hbar \nu \pm E_p$

$\vec{k}_2 = \vec{k}_1 \pm \vec{k}_{\text{photon}} \pm \vec{k}_p \sim \vec{k}_1 \pm \vec{k}_p$

* Brief Introduction to Quantum Mechanics.
Quantum particle, e.g. electron, is associated with wave function $\Psi(\vec{r}, t)$

$\Psi(\vec{r}, t)$ satisfies Schrödinger's equation:

$$\frac{i\hbar}{2m} \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi + V \Psi \tag{1}$$

Total energy $\rightarrow$ Kinetic energy $\rightarrow$ Potential energy

Energy operator: $\frac{i\hbar}{2m} \frac{\partial}{\partial t}$

Momentum operator: $-\frac{i\hbar}{\hbar} \nabla$

$|\Psi(x,t)|^2$: Probability density of finding the particle at point $x$ and time $t$.

$|\Psi|^2$: $\Psi^* \Psi$

$\int_a^b |\Psi(x,t)|^2 dx$: Probability of finding the particle between $a$ and $b$ at time $t$.

General procedure for solving Schrödinger equation:

1. Solve the time-independent Schrödinger equation:

$\Psi(x) \leftrightarrow E$

$-\frac{\hbar^2}{2m} \frac{d^2 \Psi}{dx^2} + V(x) \Psi = E \Psi$

$(3-0)$ Define Hamiltonian $\hat{H}_0 = -\frac{\hbar^2}{2m} \nabla^2 + V(\vec{r})$

$\rightarrow \hat{H}_0 \Psi = E \Psi$

Obtain eigenfunction $\Psi_n(x) \leftrightarrow E_n$

E.g. Infinite quantum well

$\Psi(x) = \frac{\sqrt{2}}{a} \sin \left( \frac{n \pi x}{a} \right)$

Quantized energy: $E_n = \frac{n^2 \hbar^2}{2ma^2}$

2. Initial wave function:

$\Psi(x, 0) = \sum_{n=1}^{\infty} C_n \Psi_n(x)$

$C_n$: obtained by matching initial conditions.

3. $\Psi(x, t) = \sum_{n=1}^{\infty} C_n \Psi_n(x) e^{-iE_n t/\hbar} = \sum_{n=1}^{\infty} C_n \Psi_n(x, t)$

Eigenfunctions are orthonormal:

$\int \Psi_i^* \Psi_j \, dx = \delta_{ij}$
Expectation value of energy: 
$$\langle H \rangle = \sum_{n=1}^{\infty} |\psi_n|^2 E_n$$