EE 529 Homework #2 (due on 4/28/2016)

1. (20%) The figure on the right shows the spontaneous emission spectrum of GaAs in thermal equilibrium at 300 K as an example.
   (a) Determine a general expression for the energy, in terms of bandgap energy $E_g$ and $k_B T$, at which the spontaneous emission spectrum $R_{sp}(E)$ reaches a peak value. Assume the semiconductor is in thermal equilibrium or under low injection/optical excitation.
   (b) Calculate its full width at half-maximum (FWHM).
   (c) Based on the result in (a), determine the peak emission wavelength if this is a GaAs LED operated at 300 K.

2. (20%) Consider direct band-to-band optical transitions in GaAs at $\lambda = 800$ nm wavelength at 300 K. The effective mass for electrons and holes are $m_e^* = 0.067m_0$ and $m_h^* = 0.52m_0$.
   (a) Find the energy levels $E_2$ and $E_1$, with respect to $E_C$ and $E_V$, for the optical transitions at this wavelength.
   (b) Calculate the value of the density of states $\rho(\nu)$ for these transitions.
   (c) By taking $\tau_{sp} = 500$ ps, find the spectral spontaneous emission rate $R_{sp}(\nu)$ for intrinsic GaAs at this optical wavelength.
   (Note: You can compare your results with Example 13.2 to check if they are reasonable.)

3. (20%) Franz-Keldysh effect in GaAs
   (a) Use what we learned about the Franz-Keldysh effect in class, plot a figure for absorption coefficient of GaAs under various electric fields like the one shown in Slide 10 of the supplementary slides. Start your plot at $E_g - \hbar \omega = 0.001$ eV. $K = 5 \times 10^4$ cm$^{-1}$ (eV)$^{-1/2}$ in GaAs. (Note: Your results will look somewhat different from the figure in Slide 10, which is obtained using a different equation.)
   (b) Calculate the tunneling probability of an electron in GaAs under an applied field of $1 \times 10^5$ V/cm when a photon of 1.4 eV is absorbed at room temperature. Compare this with the band-to-band tunneling probability of an electron in the valence band without photon absorption with the same applied field.
   (c) We want to make an optical modulator utilizing the Franz-Keldysh effect in GaAs. Assume the applied electric field is $2 \times 10^5$ V/cm, and we want to achieve 90% modulation depth, i.e., changing the output light intensity from full value to 10% of its full value. Calculate the required device length versus wavelength between 0.88 and 1 µm and plot your result in a figure.

4. (20%) (a) Use the results for the optical density of states in bulk materials and absorption coefficient in thermal equilibrium, show that the total band-to-band spontaneous recombination rate of electron-hole pairs in a semiconductor with $E_g \gg k_B T$ in thermal
equilibrium at temperature $T$ is 

$$R_{sp}^0 \approx \frac{2}{\tau_{sp}} \left( \frac{2\pi m^*_e k_BT}{\hbar^2} \right)^{3/2} e^{-\frac{E_g}{k_BT}}.$$  

(b) Use the result from (a) to show that the bimolecular recombination coefficient can be expressed by the following relation: 

$$B = \frac{1}{2\tau_{sp}} \left( \frac{2\pi k_BT}{\hbar^2} \right)^{3/2} (m^*_e + m^*_h)^{-3/2}.$$  

5. (20%) An AlGaInP/GaP LED emits at 607 nm wavelength. At this wavelength, the photopic luminous efficiency function has a value of $V(\lambda) = 0.54113$. Injection of 10 mA achieves a separation of quasi Fermi levels equal to $E_g - 0.224$ eV. The active volume of the LED is $10^5 \mu m^3$. The LED is operated at a forward bias of 2 V. For simplicity, assumes the reduced effective mass $m^*_r = 0.0594m_0$ and the spontaneous time constant $\tau_{sp} = 500$ ps. 

(a) Calculate the total output photon flux.  

(b) Assume all output photons are collected, calculate the external quantum efficiency, responsivity, and power conversion efficiency of the LED.  

(c) Find its luminous efficiency and luminous flux.