Integral Equation Based Time Domain Coupled EM-Circuit Simulation For Packaged Conductors and Dielectrics

Chuanyi Yang, **Gong Ouyang** and **Vikram Jandhyala**
Department of Electrical Engineering, Box-352500, University of Washington, Seattle, WA-98195, Telephone: 206-543-2186 Fax: 206-543-3842, jandhyala@ee.washington.edu

ABSTRACT

A full-wave time domain integral equation formulation for the simulation of finite conductors and dielectrics, linked to linear and non-linear lumped elements, is presented. The method permits coupled rigorous simulation including accounting for EMI and non-linearities in the presence of material effects. In addition, a new quadrature scheme is incorporated for exactly computing temporal delays between every section of finite basis functions defined over triangular patches. This enables finer time step resolution for non-uniform meshes than is permissible with standard Gaussian quadrature and singularity extraction.

I. Introduction

Time domain electromagnetic solvers are useful for simulating coupled circuit-electromagnetic problems involving IC packages and systems-on-chip, wherein effects of nonlinearities[1] of circuit elements can be modeled accurately. Furthermore, broadband simulation for digital and multi-frequency systems can be rendered efficient by time domain simulation. The surface-based time domain integral equation (TDIE) approach has been gaining in popularity due to its flexibility in modeling arbitrary-shaped structures and recent advance in associated fast algorithms [2].

Existing methods to couple TDIE to circuits have been based on port models, convolution methods, and the partial element equivalent circuit approach [3,4]. In this work, a generalized rigorous coupling scheme, to simultaneously simulate circuits with SPICE-like time domain simulation, and EM interactions with a TDIE method, is presented, for both conductors and dielectric materials.

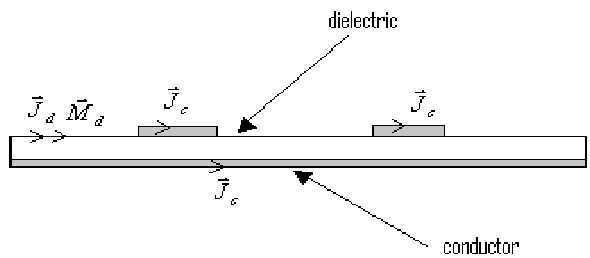

Fig. 1 Dielectric and Conductor Composite Structure

II. Formulation

The modeling of dielectrics in the circuit is facilitated by utilizing the equivalence principle [5,6]. The dielectric body is replaced by a equivalent surface S_d on which there are electric current $\mathbf{J}_d$ and magnetic current $\mathbf{M}_d$ flowing. The scattering field outside and inside of the dielectric body can be written in terms of the equivalent current $\mathbf{J}_d$ and $\mathbf{M}_d$. On the conductor surface S_c there is only electric current $\mathbf{J}_c$ flowing. Here S_d^- (S_d^+) indicates approaching S_d from outside (inside), respectively.

$$\begin{aligned} &\mathbf{E}_e^s(\mathbf{J}_c, \mathbf{J}_d, \mathbf{M}_d) + \mathbf{E}^{inc} = 0 \quad \mathbf{r} \in S_d^- \\ &\mathbf{E}_e^s(\mathbf{J}_c, \mathbf{J}_d, \mathbf{M}_d) = 0 \quad \mathbf{r} \in S_d^+ \\ &\mathbf{E}_e^s(\mathbf{J}_c, \mathbf{J}_d, \mathbf{M}_d) + \mathbf{E}^{inc} = 0 \quad \mathbf{r} \in S_c \end{aligned} \tag{1}$$

The conductor surface S_c comprises of two disjoint surfaces S_{EM} and S_{CK}, and such that on S_{EM} the standard continuity equation relating the surface current and charge holds, which is also the case on the dielectric surface S_d .

$$\nabla_s \cdot \mathbf{J}(\mathbf{r},t) + \frac{\partial \rho(\mathbf{r},t)}{\partial t} = 0 \quad \forall \mathbf{r} \in (S_{EM} \ \& \ S_d) \tag{2}$$

where ∇_s denotes the surface divergence. However, on S_{CK}, which we call *terminal* surfaces or *connecting* surfaces, the circuit current flowing onto S_{CK} from a corresponding circuit node introduces an additional source term that alters the surface current and charge on S_c. This permits connection of two disparate domains, the topology-based circuit domain, and the geometry-based EM domain.

Let S_{CK} itself be comprised of disjoint surfaces S_{CK}^m $m = 1,.....,M$. Each such unique sub-surface S_{CK}^m is termed one of *M terminals*. On S_{CK}^m the modified continuity equation has the following form

$$\nabla_s \cdot \mathbf{J}(\mathbf{r},t) + \frac{\partial \rho(\mathbf{r},t)}{\partial t} = I_c^m(\mathbf{r},t) \quad \forall \mathbf{r} \in S_{CK}^m \quad m = 1,.....,M \tag{3}$$

where $I_c^m(\mathbf{r},t)$ denotes the scalar volumetric current density produced on S_{CK}^m via a circuit interconnect.

The current density introduced by the circuit interconnection produces an additional source or sink of charge that alters the time dependent scalar potential and the resulting the scattering electric field from S_c and S_d^-.

$$\mathbf{E}^s(\mathbf{r},t) = -\frac{\partial \mathbf{A}(\mathbf{J}_c,\mathbf{J}_d,\mathbf{r},t)}{\partial t} - \nabla\Phi_{EM}(\mathbf{J}_c,\mathbf{J}_d,\mathbf{r},t) - \nabla\Phi_{CK}(I_c,\mathbf{r},t) - \frac{1}{\varepsilon_e}\nabla\times\mathbf{F}(\mathbf{M}_d,\mathbf{r},t) \quad \forall \mathbf{r} \in (S_{EM} \;\&\; S_d^-) \tag{4}$$

The scattering field from S_c and S_d^- can be written in term of magnetic vector potential, **A,** electric scalar potential, Φ, and electric vector potential **F** **w**here $\nabla\Phi_{EM}(\mathbf{J}_c,\mathbf{J}_d,\mathbf{r},t)$ is the contribution from the EM current, $\nabla\Phi_{CK}(I_c,\mathbf{r},t)$ is the contribution from the circuit current, and ε_e is the permitivity of the medium exterior to the dielectric body.

The scattering field from S_d^+ can be written as following

$$\mathbf{E}^s(\mathbf{r},t) = -\frac{\partial \mathbf{A}(\mathbf{J}_d,\mathbf{r},t)}{\partial t} - \nabla\Phi(\mathbf{J}_d,\mathbf{r},t) - \frac{1}{\varepsilon_d}\nabla\times\mathbf{F}(\mathbf{M}_d,\mathbf{r},t) \quad \forall \mathbf{r} \in S_d^+ \tag{5}$$

where ε_d is the permitivity of the medium interior to the dielectric body. In addition to the scattering field, two more conditions are required for the circuit interconnection: electrically small terminals are assumed to be equipotential, leading to

$$\Phi_{EM}(\mathbf{J}_c,\mathbf{J}_d,\mathbf{r},t) + \Phi_{CK}(I_c,\mathbf{r},t) = V_m(t) \tag{6}$$

for $m = 1,.....,M$ where $V_m(t)$ is the circuit potential at the circuit node connected to the terminal *m*, and the final set of self-consistency equations relates to the application of Kirchoff's Current Law at the *M* nodes connected to the terminals.

$$\sum_{j=1}^{adj(m)} i_j^m(t) = \int_{S_{CK}^m} I_c^m(t)\,ds \tag{7}$$

The system of equations can be combined to yield the time-domain circuit-EM coupled system. The linear and non-linear circuits connected to the terminals are modeled by Modified Nodal Analysis (MNA). The coupled system has the form, (8)

$$\begin{bmatrix} \overline{\mathbf{Z}_{JJ0}^{cc}} & \overline{\mathbf{Z}_{JJ0}^{cd}} & \overline{\mathbf{Z}_{JM0}^{cd}} & \overline{\mathbf{Q}_{JI0}^{cc}} & \bar{\mathbf{0}} \\ \overline{\mathbf{Z}_{JJ0}^{dc}} & \overline{\mathbf{Z}_{JJ0}^{dd}} & \overline{\mathbf{Z}_{JM0}^{dd}} & \overline{\mathbf{Q}_{JI0}^{dc}} & \bar{\mathbf{0}} \\ \bar{\mathbf{0}} & \overline{\mathbf{Z}_{MJ0}^{dd}} & \overline{\mathbf{Z}_{MM0}^{dd}} & \bar{\mathbf{0}} & \bar{\mathbf{0}} \\ \overline{\mathbf{Z}_{IJ0}^{cc}} & \overline{\mathbf{Z}_{IJ0}^{dd}} & \bar{\mathbf{0}} & \overline{\mathbf{Q}_{II0}^{cc}} & \overline{\mathbf{C}} \\ \bar{\mathbf{0}} & \bar{\mathbf{0}} & \bar{\mathbf{0}} & \overline{\mathbf{C}^{\mathbf{T}}} & \overline{\mathbf{MNA}_0} \end{bmatrix} \begin{bmatrix} \mathbf{J_c}(t_j) \\ \mathbf{J_d}(t_j) \\ \mathbf{M_d}(t_j) \\ \mathbf{I_c}(t_j) \\ \mathbf{CKT}(t_j) \end{bmatrix} = \sum_{i=1}^{j} \begin{bmatrix} \overline{\mathbf{Z}_{JJi}^{cc}} & \overline{\mathbf{Z}_{JJi}^{cd}} & \overline{\mathbf{Z}_{JMi}^{cd}} & \overline{\mathbf{Q}_{JIi}^{cc}} & \bar{\mathbf{0}} \\ \overline{\mathbf{Z}_{JJi}^{dc}} & \overline{\mathbf{Z}_{JJi}^{dd}} & \overline{\mathbf{Z}_{JMi}^{dd}} & \overline{\mathbf{Q}_{JIi}^{dc}} & \bar{\mathbf{0}} \\ \bar{\mathbf{0}} & \overline{\mathbf{Z}_{MJi}^{dd}} & \overline{\mathbf{Z}_{MMi}^{dd}} & \bar{\mathbf{0}} & \bar{\mathbf{0}} \\ \overline{\mathbf{Z}_{IJi}^{cc}} & \overline{\mathbf{Z}_{IJi}^{dd}} & \bar{\mathbf{0}} & \overline{\mathbf{Q}_{IIi}^{cc}} & \bar{\mathbf{0}} \\ \bar{\mathbf{0}} & \bar{\mathbf{0}} & \bar{\mathbf{0}} & \bar{\mathbf{0}} & \overline{\mathbf{MNA}_i} \end{bmatrix} \begin{bmatrix} \mathbf{J_c}(t_{j-i}) \\ \mathbf{J_d}(t_{j-i}) \\ \mathbf{M_d}(t_{j-i}) \\ \mathbf{I_c}(t_{j-i}) \\ \mathbf{CKT}(t_{j-i}) \end{bmatrix} + \begin{bmatrix} \mathbf{SRC_{EM}^c}(t_j) \\ \mathbf{SRC_{EM}^d}(t_j) \\ \bar{\mathbf{0}} \\ \bar{\mathbf{0}} \\ \mathbf{SRC_{CK}}(t_j) \end{bmatrix} \tag{8}$$

The vector $\mathbf{SRC_{EM}^c}(t_j)$ represents the tested incident field, and the $\mathbf{SRC_{CK}}(t_j)$ denotes the values of circuit sources. The matrix $\overline{\mathbf{C}}$ is a sparse bipolar adjacency matrix that is used for enforcing Kirchoff's Voltage and Current Laws at the circuit nodes connected to the terminals. This approach enables both linear and non-linear (through local Newton-Raphson on the MNA sub-matrix) circuit simulation in conjunction with EM simulation.

III. Stability and Accuracy Enhancement

A new integration scheme in polar coordinates which significantly improves the late time stability associated with TDIE and accuracy as well is implemented, for realistic non-uniform meshes. The new formulation takes the exact delay difference into account between every section of two interacting basis functions and uses the polar coordinate integration scheme for each patch-patch interaction calculation. And this method is exact in the sense that all source points are modeld with exact retarded time. The advantage of this technique is the ability to reduce the smallest time step in an implicit scheme beyond what is possible by standard Gaussian quadrature and singularity extraction methods. Figure 2 demonstrates the smallest time step possible with the standard method (old) and the proposed polar coordinate quadrature (new). The proposed approach has been incorporated into a TDIE scheme and is particularly useful for non-uniform mesh examples.

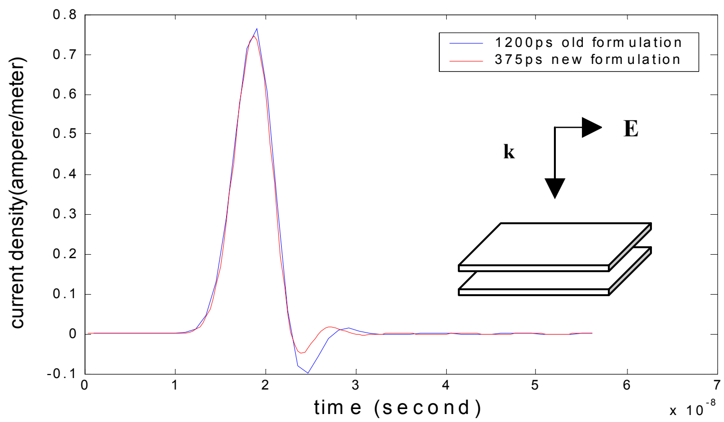

Fig.2 Stability Comparison Between New and Old Formulation

IV. Simulation Result

The ACES IEEE/EMC Society TC9 Challenging Problems have been designed to challenge various modeling techniques. One of the 2001 Challenging Problems "Differential Pair Over Split Ground"(Fig. 3) is used to test the coupled solver. In this example, a ground plane with or without split is located on the left hand side. A driver consisting of two trapezoidal voltage sources with rising and falling time of 200ps, roof time of 1000ps and zero internal impedance, delivers a differential mode signal to two symmetric lines. For the split ground plane case, a decoupling capacitor may be placed across the split. The decoupling capacitor is equivalent to a series connection of a 10nf capacitor, 10nh inductor and 0.03ohm resistor. The pair of wires extends for 1 meter and the ends of the differential lines are terminated by a 100ohm resistor. The task is the find the voltage between Line+ and CM ground point in three case, (1) no split in plane, (2) split in plane and no decoupling capacitor, (3) split in plane with decoupling capacitor. Fig. 4 and Fig. 5 are the simulation result with or without the dielectric substrate that match well with results in the available literature.

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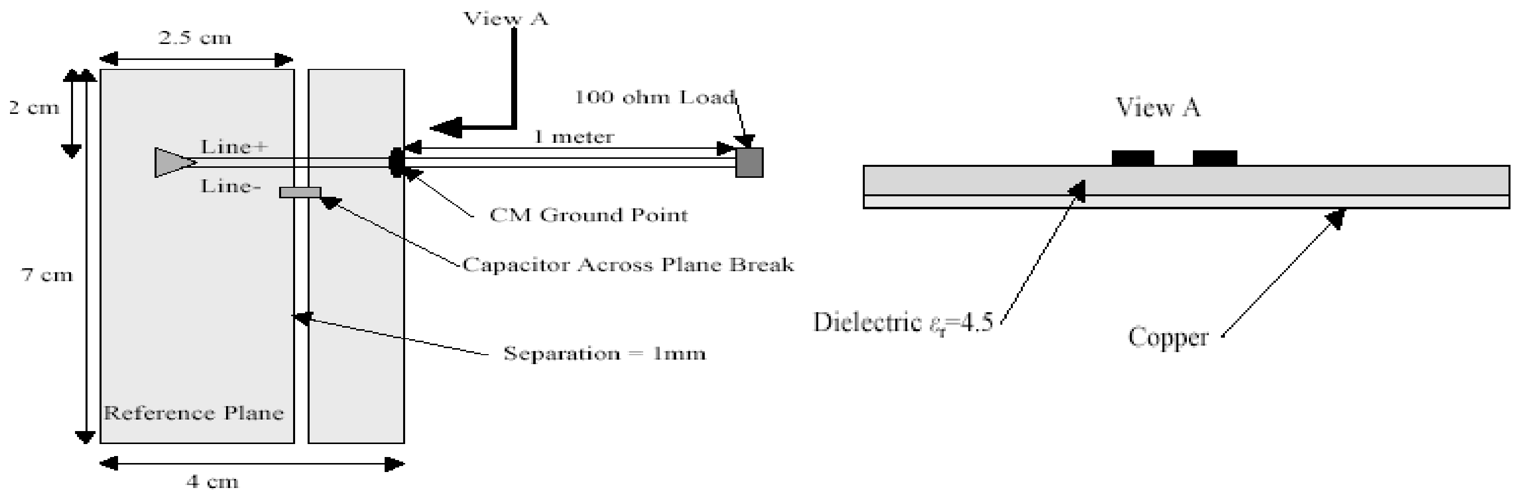

Fig. 3 Differential Pair Over Split ground plane

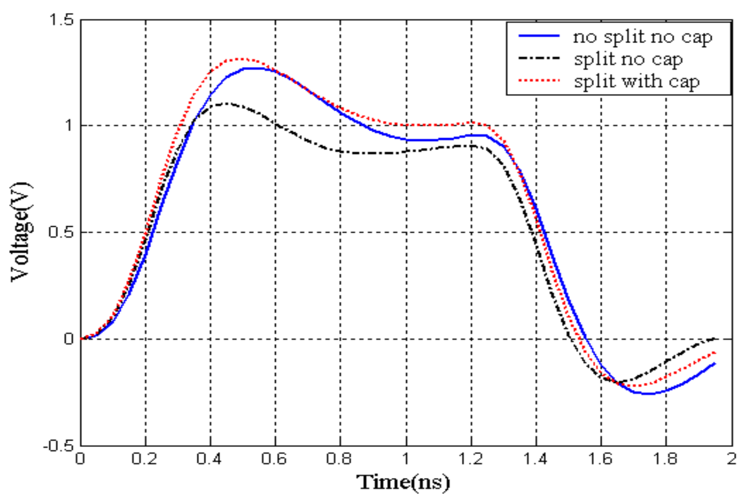

Fig. 4 Line+ to CM voltage(with dielectric)

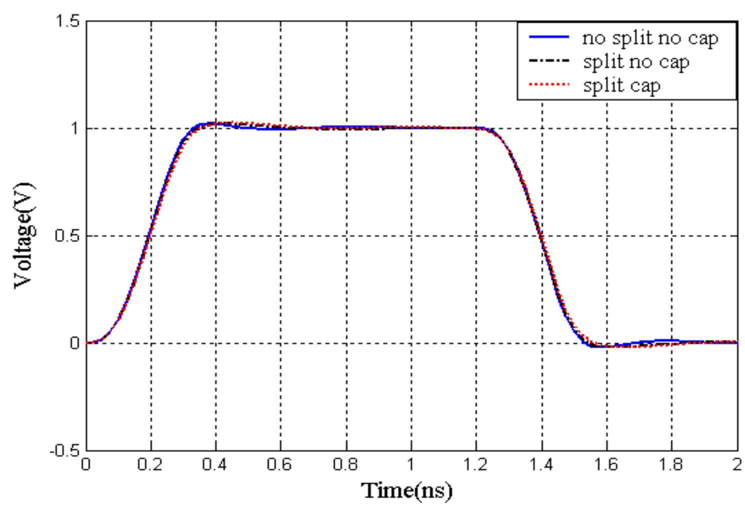

Fig. 5 Line+ to CM voltage(without dielectric)