Coded Asynchronous CDMA and Its Efficient Detection

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***Abstract*— In this paper, receiver design and performance analysis for *coded* asynchronous code-division multiple access (CDMA) systems is considered. The receiver front-end consists of the near–far resistant multiuser detector known as the projection receiver (PR). The PR performs multiple-access interference resolution and is followed by error-control decoding. The output of the projection receiver yields the appropriate metric (i.e., soft information) for decoding of the coded sequences. An expression for the metric is derived that allows the use of a standard sequence decoder (e.g., Viterbi algorithm, M-algorithm) for the error-control code. It is then shown that the metric computer has an elegant adaptive implementation based on an extension of the familiar recursive least squares (RLS) algorithm. The adaptive PR operates on a single sample per chip and achieves a performance virtually identical to the algebraic PR, but with significantly less complexity. The receiver performance is studied for CDMA systems with fixed and random spreading sequences, and theoretical performance degradations with regard to the single-user bound are derived. The near–far resistance of the PR is also proven, and demonstrated by simulation.**

***Index Terms*— CDMA, error control coding, least means squares (LMS), multiple access, multiuser receivers, projection receiver, recursive least means squares (RLS).**

I. Introduction

IN code-division multiple-accessing (CDMA) systems, multiple users transmit simultaneously and independently over a common channel using preassigned spreading waveforms or signature sequences that uniquely identify the users [1], [2]. If the *received* waveforms corresponding to the active users are orthogonal over the symbol interval, the conventional receiver consisting of a bank of matched-filters/correlators provides optimum performance. The problem of design of strictly orthogonal codes for a large number of users (relative to the processing gain) is known to be difficult for the synchronous case; the practical reality of asynchronous transmission renders this pursuit almost futile. Hence, nonorthogonal spreading waveforms with low crosscorrelation properties such as pseudorandom sequences [3], [4] are employed in practice. In such nonorthogonal CDMA systems, the conventional correlation receiver suffers from two major drawbacks. First, strict power control is required to alleviate the so-called near–far problem [2], a situation where the signal of a strong nearby user prevents detection of users that are further away and are received with low power. Second, the multiuser interference starts to significantly degrade[1] the performance of the conventional detector for uncoded signals when the number of active users exceeds about 10% of the processing gain (see also, e.g., [2]).

Uncoded multiuser detectors which treat all signals as information bearing and decode such users jointly have been studied in the literature to some extent [5]–[16]. The major promise of such multiuser detectors is in enhancing spectral utilization and a relaxation of the need for precise power control so vitally important for the conventional detector. Receivers whose performance is largely unaffected by power variations of other users are called *near–far resistant*. Optimal detection of asynchronous CDMA is theoretically possible [5], but its practical realization is only feasible for very small numbers of users due to the large complexity of such detectors [17], which grows exponentially with the number of users. In practice, suboptimal, low-complexity receivers such as linear (matrix-based) detectors are possible candidates for implementation in future digital CDMA systems in several wireless applications.

Combining forward error-control (FEC) coding with CDMA is a relatively new approach that has been studied only recently [18]–[22]. Often, FEC systems are simply concatenated with the multiuser detector using heuristic arguments. A general error-control coding paradigm recommends against making any *intermediate hard decisions*. Hence, the proper transfer of appropriate "soft" information between the receiver stage that performs multiuser resolution and subsequent FEC decoder is vitally important. In this work, we employ the Projection Receiver (PR) developed in [14] and [15] as the receiver first stage for multiuser interference cancellation in a structured manner, followed by decoding of the FEC code. The resultant class of receivers is ideally near–far resistant (a property it shares with the decorrelator), i.e., we show that the power levels of the interfering users have no influence on performance. The PR can be tailored to achieve complexity–performance tradeoffs between the optimal detector at the high end and the standard decorrelator (zero-forcing) detector at the low end.

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[1] An estimate of the degradation can be found by assuming that the interferers act as noise, hence the interfering energy per bit is approximated by KE_b/N, and $P_b \approx Q(\sqrt{N/K})$.

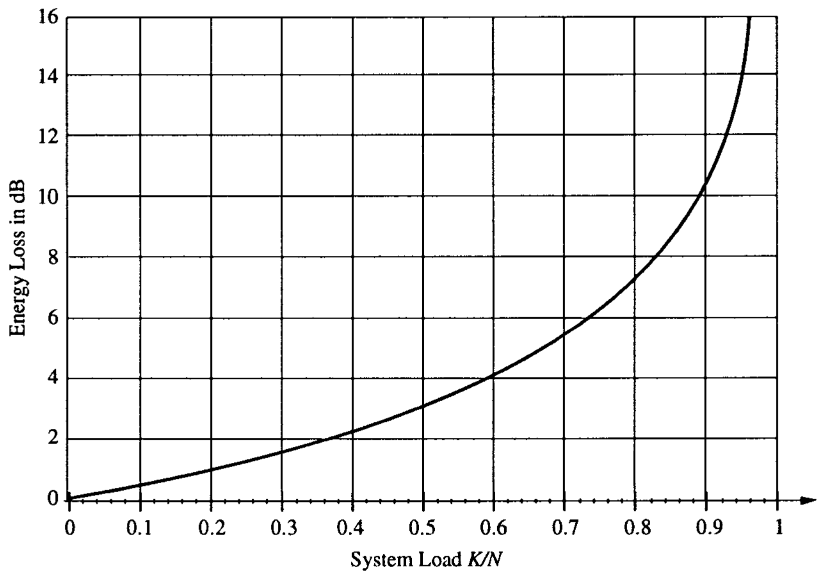

Fig. 1. Loss factor of the projection receiver.

This paper is organized as follows. In Section II we present a synopsis of our main results, in Section III we introduce the CDMA system model and our linear algebraic model is derived. The linear algebraic model makes the discussion of the system significantly easier. The first half of Section IV discusses the metric generation, and the second half shows how the system passes the metric to the sequence estimator. Section IV, entitled "Algebraic Projection Receiver" deals with the algebraic system model and its underlying geometry. Section V presents the adaptive generation of the metrics, i.e., the adaptive projection receiver. Section VI presents simulation results, and in Section VII we analyze the performance of the PR for random spreading sequences by comparing the performance of the PR to the single-user bound (the performance of the FEC system in the absence of interferers). We do this by calculating degradation factors with respect to (w.r.t.) the single-user bound. The accuracy of our calculations is demonstrated by comparing the analytical degradation results with those of the simulated performance. Finally, Section VII contains our concluding remarks.

II. Main Results

The projection receiver (PR) is a near–far resistant multiuser receiver that cancels the interference statistically, i.e., without explicitly detecting the transmitted data of the interfering users. The PR utilizes a linear preprocessor which projects the received signal onto the signal subspace orthogonal to the signal space "contaminated" by the interference. The projection operation results in an "effective energy loss" $\mathcal{L}$ (in decibels) with respect to an interference-free reference system, i.e., in the presence of multiaccess interference, an amount $\mathcal{L}$ of extra energy needs to be expended to achieve the performance of the interference-free reference system.

This energy loss is given by

$$\mathcal{L} \approx 10 \log_{10} \left(\frac{N}{N-K+1} \right) \quad \text{dB} \tag{1}$$

where N is the processing gain and K is the number of simultaneous asynchronous CDMA users. This loss factor is plotted in Fig. 1 as a function of K/N. It is remarkable that $\mathcal{L} \leq 3$ dB for $K \leq N/2$, i.e., even with a 50% system load only 3-dB excess energy is required. Equation (1) is proven as a lower bound for random spreading codes in Section VIII, while the simulations in Section VII demonstrate its achievability for both random spreading sequences and fixed spreading sequences (using Gold codes).

III. CDMA System Model

In a CDMA multiuser system, K users access the same channel each using a unique spreading sequence of duration equal to the symbol interval T. The signal at the receiver input in the presence of additive, white Gaussian noise[2] $n(t)$, is given by

$$s(t) = \sum_{j=1}^{L} \sum_{k=1}^{K} d_j^{(k)} \sqrt{w_j^{(k)}} c_j^{(k)}(t - jT - \tau^{(k)}) + n(t) \tag{2}$$

where $c_j^{(k)}(t)$ is the signature waveform of user k during the transmission of the jth symbol $d_j^{(k)}$ assumed to be binary $\{-1, 1\}$, $w_j^{(k)}$ is the energy of the jth symbol sent by the kth user, and $\tau^{(k)}$ is the delay for the kth user. The unit-energy signature waveform $c_j^{(k)}(t)$ for the kth user is given by

$$c_j^{(k)}(t) = \sum_{n=0}^{N-1} \alpha_{(j-1)N+n}^{(k)} p(t - nT_c) \tag{3}$$

where $\alpha_{(j-1)N+n}^{(k)} \in \{-1, +1\}$ are the symbols of the discrete spreading sequences, and the chip waveform $p(t)$ is taken to be a rectangular pulse for the remainder of this paper. The processing gain (equivalently, number of chips per symbol) is

[2] We concentrate on the additive white Gaussian noise (AWGN) model in this paper. Extensions to fading and fading multipath channels are possible using approaches analogous to those presented in this paper.

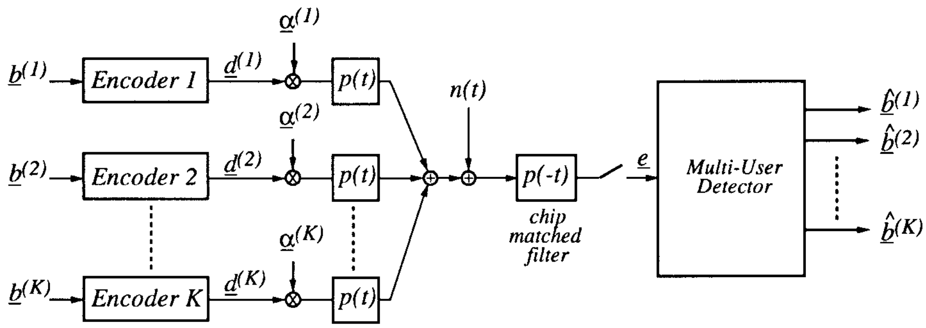

Fig. 2. Baseband block diagram of a multiuser CDMA communications system using forward error coding (FEC).

assumed identical for all users and equal to N, and each user transmits a sequence of L symbols. For simplicity, we further assume that the delays $\tau^{(i)}$ in (2) are integer multiples of the chip duration T_c, i.e., the system is *chip* synchronous. Note that the symbol synchronous case is simply a special case of (2) with $\tau^{(i)} = 0, \forall i$.

Fig. 2 shows the basic (baseband) system block diagram of a CDMA multiuser system using error-control coding. The samples after the chip matched filter are spaced by multiples of T_c, i.e., we are considering a single-sample per chip system.

Each user's data stream $\underline{b}^{(k)} = [b_1^{(k)}, \cdots, b_{L_u}^{(k)}]$ drives an encoder of rate $R = L_u/L$, whose outputs $\underline{d}^{(k)} = [d_1^{(k)}, \cdots, d_L^{(k)}]^T$ are spread by $\underline{\alpha}^{(k)} = \alpha_0^{(k)}, \cdots, \alpha_{LN-1}^{(k)}$, the spreading sequence of the kth user, and modulated chip-wise by $p(t)$. The encoder can be a block or convolutional encoder which is terminated suitably at the end of the transmission interval.

Two scenarios—random and deterministic signature sequences—are considered. In the former case, the spreading sequences are generated by taking $\underline{\alpha}^{(k)}$ to be long m-sequences [2] with different phases for different users. In the latter case, Gold codes [4] of duration T were used as spreading sequences by periodic repeating in each symbol interval, i.e., $\alpha_{jN+n}^{(k)} = \alpha_n^{(k)}, \forall j$. As will be seen, the performance of these two systems can differ significantly. The section of the spreading sequence during symbol j is denoted by

$$\underline{\alpha}_j^{(k)} = \left[\alpha_{(j-1)N}^{(k)}, \cdots, \alpha_{jN-1}^{(k)}\right]. \qquad (4)$$

The receiver consists of a chip-matched filter sampled at multiples of the chip interval T_c. The vector $\underline{e}$ of sample values has dimension $LN \times 1$. It is processed by the multiuser decoder which produces the bit-stream estimates $\hat{\underline{b}}^{(k)}$. The vector $\underline{e} = [\underline{e}_1^T, \underline{e}_2^T, \cdots, \underline{e}_L^T]^T$ can now be described by the linear model

$$\underline{e} = \mathcal{A}\mathcal{W}\underline{d} + \underline{n} \qquad (5)$$

where $\underline{e}_j^T = [e_{jN+1}, \cdots, e_{(j+1)N}]$ is the N-vector of samples during the jth interval of duration $T = NT_c$, $\underline{d} = [\underline{d}_1^T, \underline{d}_2^T, \cdots, \underline{d}_L^T]^T$, and $\underline{d}_j^T = [d_j^{(1)}, \cdots, d_j^{(K)}]$ is the vector of encoded symbols at time j. The noise $n(t)$ is additive white Gaussian (AWGN) with one-sided noise power spectral density N_0. The vector $\underline{n}$ of noise samples is white with variance $N_0/2$ per chip sample. $\mathcal{W}$ is a diagonal matrix of dimension LK determined by (square root of) the user energies, i.e.,

$$\operatorname{diag}(\mathcal{W}) = \left[\sqrt{w_1^{(1)}}, \cdots, \sqrt{w_j^{(k)}}, \cdots, \sqrt{w_L^{(K)}}\right]. \qquad (6)$$

The matrix

$$\mathcal{A} = [\underline{a}_1^{(1)}, \cdots, \underline{a}_L^{(1)}, \cdots, \underline{a}_1^{(K)}, \cdots, \underline{a}_L^{(K)}]$$

where the columns of $\mathcal{A}$ consist of spreading sequences shifted appropriately, such that $\alpha_j^{(k)}$ starts at position $(j-1)T + \tau_j$, i.e.,

$$\underline{a}_j^{(k)} = [0, \cdots, 0, \alpha_{(j-1)N}^{(k)}, \cdots, \alpha_{jN-1}^{(k)}, 0, \cdots, 0].$$

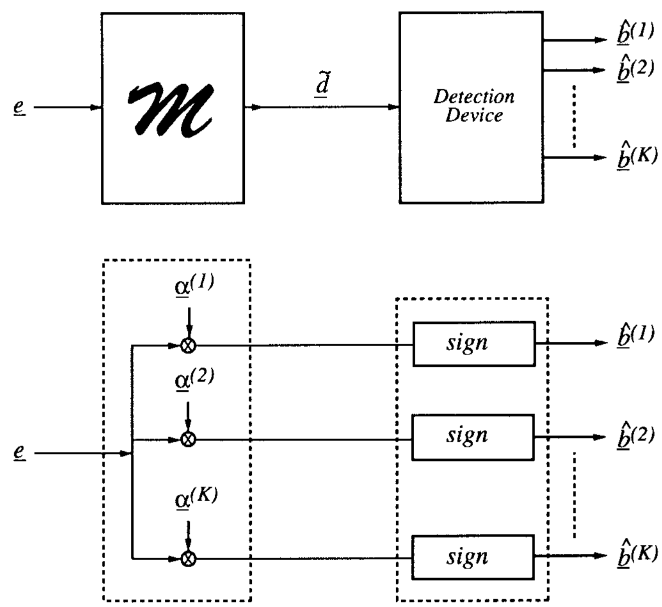

Fig. 3. Generic linear multiuser receiver and the correlation receiver.

A linear multiuser receiver is now simply described by a $KL \times NL$ matrix $\mathcal{M}$ which produces soft estimates $\tilde{\underline{d}}$ from $\underline{e}$, i.e.,

$$\tilde{\underline{d}} = \mathcal{M}\underline{e} \qquad (7)$$

followed by a decision device. The generic structure of such a linear receiver is shown in Fig. 3.

The conventional correlation receiver is of that flavor, whereby $\mathcal{M} = \mathcal{A}^T$, and the decision device is a symbol-by-symbol threshold operation.

IV. Algebraic Projection Receiver

A. Linear Metric Generation

Since the noise samples in (5) are Gaussian, the maximum-likelihood estimate of $\underline{d}$ is given by

$$\hat{\underline{d}}^{(\mathrm{MLSE})} = \arg\min_{\underline{d}\in\mathcal{D}} |\underline{e} - \mathcal{A}\mathcal{W}\underline{d}|^2 \tag{8}$$

where $\mathcal{D}$ denotes the set of sequence candidates $\underline{d}$ taken from $\{-1, +1\}$, and constrained by the error-control codes. It is quickly seen that an exhaustive search through the candidates in (8) is computationally prohibitive, even for moderate numbers of users. The projection receiver, originally proposed in [14] and [15], reduces complexity by partitioning $\underline{d}$ into two sets

$$\underline{d} \rightarrow \{\underline{d}_u, \underline{d}_c\}. \tag{9}$$

The complexity reduction is now achieved by estimating $\underline{d}_u$ over the real (unconstrained) domain, and estimating only $\underline{d}_c$ over the constrained domain, now denoted by $\mathcal{D}_c$. This leads to the partitioned minimization

$$\hat{\underline{d}}^{(\mathrm{PR})} = \arg\min_{\underline{d}_c\in\mathcal{D}_c} \min_{\underline{d}_u} |\underline{e} - \mathcal{A}\mathcal{W}\underline{d}|^2. \tag{10}$$

The constrained portion $\underline{d}_c$ of the data can be any combination of transmitted symbols, but typically $\underline{d}_c$ would be the sequence of a single user, i.e., $\underline{d}_c = \underline{d}^{(k)} = [d_1^{(k)}, \cdots, d_L^{(k)}]$, or that of a few "desired" users. The complexity savings of (10) over (8) are substantial. In the *fully projected* case, where $\underline{d}_c$ contains the symbols of only one user, the savings over maximum-likelihood decoding are a factor of 2^K!

Rewriting (5) in terms of the constrained (desired) and unconstrained (undesired) user

$$\underline{e} - \mathcal{A}_c\mathcal{W}_c\underline{d}_c = \mathcal{A}_u\mathcal{W}_u\underline{d}_u + \underline{n} \tag{11}$$

where $\mathcal{A}$ has been conformably partitioned into the components $\mathcal{A}_c$ corresponding to the symbols of the desired users $\underline{d}_c$ while $\mathcal{A}_u$ represent the columns corresponding to the symbols $\underline{d}_u$ from the undesired users (the diagonal matrix $\mathcal{W}$ is partitioned likewise). The minimization over $\underline{d}_u$ is quadratic and can be accomplished in closed form [15] to yield

$$\hat{d}_u = \left(\mathcal{A}_u^T\mathcal{A}_u\right)^{-1}\mathcal{A}_u^T(\underline{e} - \mathcal{A}_c\mathcal{W}_c\underline{d}_c) \tag{12}$$

for the joint estimate[3] of

$$\sqrt{w_j^{(l)}}\,d_j^{(l)}, \qquad l \neq k$$

of the unconstrained symbols.

$$\hat{\underline{d}}_c^{(\mathrm{PR})} = \arg\min_{\underline{d}_c\in\mathcal{D}_c} |\mathcal{M}(\underline{e} - \mathcal{A}_c\mathcal{W}_c\underline{d}_c)|^2 \tag{13}$$

where

$$\mathcal{M} = \mathcal{I} - \mathcal{A}_u\left(\mathcal{A}_u^T\mathcal{A}_u\right)^{-1}\mathcal{A}_u^T. \tag{14}$$

The matrix $\mathcal{M}$ is the projection matrix onto the nullspace of $\mathcal{A}_u^T$, hence the name "projection receiver." Note also that the powers $\mathcal{W}_u$ of the unconstrained users are *not* required by the detector.[4]

The projection front-end of the receiver produces a metric

$$\Lambda(\underline{d}_c) = |\mathcal{M}(\underline{e} - \mathcal{A}_c\mathcal{W}_c\underline{d}_c)|^2 \tag{15}$$

for a given hypothesized sequence $\underline{d}_c$. The detector will now choose as hypothesis the sequence $\hat{\underline{d}}_c$ with the smallest metric $\Lambda(\hat{\underline{d}}_c)$.

B. Geometric Interpretation

In this subsection we present a geometric interpretation of the metric generation (15). The matrix (14) is the projection onto the space orthogonal to that spanned by the columns in $\mathcal{A}_u$.

The hypothesis for a candidate sequence $\underline{d}_c$ is produced by subtracting its effect from the received sequence $\underline{e}$, i.e., we generate $\underline{e} - \mathcal{A}_c\mathcal{W}_c\underline{d}_c$. This vector is then projected using the projection matrix $\mathcal{M}$. The length of the resulting projected vector is the reliability metric for the given sequence. What the receiver does, in effect, is that it ignores the contributions which are "contaminated" by the interferers. This results in a shortening of the received signal vector (since $|\mathcal{M}\underline{x}| \leq |\underline{x}|$), which is equivalent to an energy loss. In Section VII we will calculate this energy loss and see that it is not that severe unless the system is very highly loaded with users and the orthogonal complement of the interferer space has a very small dimensionality. It also becomes obvious that the PR is ideally near–far resistant. Since it operates in the space orthogonal to the interference, the power of the interference is irrelevant.

C. Recursive Sequence Detection

Testing the metrics (15) for all possible sequences $\underline{d}_c$ of length L is still a formidable task. However, this metric calculation can be accomplished in a recursive fashion. To this end let us partition $\mathcal{M}$ into $N \times N$ blocks. $\mathcal{M}$ is a full matrix, but its off-diagonal terms decrease rapidly with distance from the diagonal. This prompts us to work with a block-diagonal approximation $\mathcal{M}^{(s)}$ of $\mathcal{M}$ retaining s of the off-diagonal blocks on either side of the diagonal. So, for example, for $s = 1$

$$\mathcal{M}^{(1)} = \begin{bmatrix} M_{11} & M_{12} & 0 & \cdots & 0 \\ M_{21} & M_{22} & M_{23} & & \vdots \\ 0 & M_{32} & M_{33} & M_{34} & \vdots \\ \vdots & & & \ddots & \vdots \\ 0 & \cdots & & & M_{LL} \end{bmatrix}. \tag{16}$$

Note that due to (14), $M_{ij} = M_{ji}^T$. The choice of s is not obvious and would need to be determined by experiment. Since we will not pursue this receiver much further, suffice it to mention that we found that typical values of s are about $1 \leq s \leq 3$.

In what follows we shall concentrate on the fully projected case, i.e., the constrained symbols are those of a single user

[3] Note that this estimate is the decorrelator estimate of the received signal with the hypothesis $\underline{d}_c$ removed.

[4] In fact, if we assume that those powers are unknown, (10) is the maximum-likelihood (ML) detector for $\underline{d}$.

only, denoted by $\underline{d}^{(k)}$. The sequence metric in (15) can now be written as

$$\begin{aligned}\Lambda(\underline{d}^{(k)}) &= \left|\mathcal{M}\left(\underline{e} - \sum_{j=1}^{L} \underline{a}_j^{(k)} w_j^{(k)} d_j^{(k)}\right)\right|^2 \\ &= \left(\left(\underline{e} - \sum_{j=1}^{L} \underline{a}_j^{(k)} w_j^{(k)} d_j^{(k)}\right)^T \right. \\ &\qquad \left. \times\, \mathcal{M}^T\mathcal{M}\left(\underline{e} - \sum_{j=1}^{L} \underline{a}_j^{(k)} w_j^{(k)} d_j^{(k)}\right)\right) \end{aligned} \tag{17}$$

and, since $\mathcal{M}$ is a projection matrix it is idempotent, i.e., $\mathcal{M}^T\mathcal{M} = \mathcal{M}$, where transposition has no effect since $\mathcal{M}$ is also symmetric. We may simplify (17) further by neglecting terms common to all sequences $\underline{d}^{(k)}$, and obtain a recursive form, given by

$$\Lambda(\underline{d}^{(k)}) = \sum_{j=1}^{L} \lambda_j(\underline{d}^{(k)}) \tag{18}$$

where

$$\begin{aligned}\lambda_j(\underline{d}^{(k)}) = &- \sum_{i=j-s}^{j-1} \left(\underline{e}_i^T M_{ij}\right) \underline{a}_j^{(k)} w_j^{(k)} d_j^{(k)} \\ &- \underline{e}_j^T \sum_{i=j-s}^{j} \underline{e}_j^T M_{ji} \underline{a}_i^{(k)} w_i^{(k)} d_i^{(k)} \\ &+ w_j^{(k)} d_j^{(k)} \underline{a}_j^{(k)T} \sum_{i=j-s}^{j-1} M_{ji} \underline{a}_i^{(k)} w_i^{(k)} d_i^{(k)}. \end{aligned} \tag{19}$$

The above metric is reminiscent of the case of a point-to-point channel with intersymbol interference (ISI) [23], where the interference by adjacent symbols is created by the asynchronicity of the CDMA channel.

For synchronous CDMA, the metrics in (19) simplify to

$$\lambda_j(\underline{d}^{(k)}) = \underline{e}_j^T M_{jj} \underline{a}_j^{(k)} w_j^{(k)} d_j^{(k)} \tag{20}$$

since $M_{ij} = 0$ for $i \neq j$. The synchronous case is treated in [14]–[16].

Detection via (19) requires a sequence detector. In our case of using an error-control code,[5] this results in an augmentation of the code state space, similar to the case of coding for ISI channels [25]. The augmented code space will contain 2^s times as many states as the original code state space. This is the price to be paid by the asynchronous nature of the channel. The augmented code state space can now be searched by a trellis search algorithm, e.g., the Viterbi algorithm, or the M-algorithm [24]. Furthermore, reduced-complexity partial searches [26], [27] might be used to decrease complexity. If the metrics are calculated according to (19), we refer to the decoder as the *Algebraic Projection Receiver*.

[5] We assume that an error-control code which can be represented by a trellis is used. The most obvious such choice would be a convolutional code. However, block codes, too, have a code trellis [24], which can be used.

V. Adaptive Projection Receiver

A. Adaptive Metric Generation

The metric calculation according to the algebraic projection receiver requires the calculation of the matrices M_{ij} at each symbol interval j. For a time-varying CDMA system, i.e., a system where different spreading sequences are used in different symbol intervals, this poses a significant computational burden. In this section we discuss a recursive procedure which avoids the matrix inversions required for the algebraic metric calculation. Recalling the sequence metric (15) and the definition of the projection matrix (14) we can write

$$\begin{aligned}\Lambda(\underline{d}_c) &= |\mathcal{M}(\underline{e} - \mathcal{A}_c\mathcal{W}_c\underline{d}_c)|^2 \\ &= |\underline{e} - \mathcal{A}_c\mathcal{W}_c\underline{d}_c - \mathcal{A}_u\hat{\underline{d}}_u|^2 \\ &= |\tilde{\underline{e}} - \mathcal{A}_u\hat{\underline{d}}_u|^2 \end{aligned} \tag{21}$$

where $\tilde{\underline{e}} = \underline{e} - \mathcal{A}_c\mathcal{W}_c\underline{d}_c$. The estimate $\hat{\underline{d}}_u$ for the unconstrained symbols is obtained as the least squares (LS) solution to

$$\min_{\underline{d}_u} |\tilde{\underline{e}} - \mathcal{A}_u\underline{d}_u|^2 = \min_{\underline{d}_u} |\tilde{\underline{e}} - \hat{\underline{e}}|^2. \tag{22}$$

The exact LS solution to (22) is given by (12), however, attempting to avoid its algebraic evaluation, we propose to use an adaptive generation of $\underline{\mathcal{A}_u\hat{d}_u}$ to be used in (21). It is based on the recursive least squares (RLS) algorithm [29], [28], and recursively generates

$$\hat{\underline{e}}(i) = \sum_{j=1}^{L} \sum_{\substack{l=1\\(l\neq k)}}^{K} v_j^{(l)}(i-1)\underline{a}_j^{(l)}(i) \tag{23}$$

where $v_j^{(l)}(i-1)$ is the current estimate of $d_j^{(l)}$, and the index i is the chip index running from $1 \to LN$. $\hat{\underline{e}}(i)$ and $\underline{a}_j^{(l)}(i)$ denote the ith component of the vectors $\hat{\underline{e}}$ and $\underline{a}_j^{(l)}$, respectively.

The standard LS solution to (22) at chip index n is given by [29, p. 479]

$$\boldsymbol{\Phi}(n)\boldsymbol{v}(n) = \boldsymbol{\theta}(n) \tag{24}$$

where

$$\boldsymbol{v}(n) = \left[\boldsymbol{v}_1^T(n), \cdots, \boldsymbol{v}_L^T(n)\right] \tag{25}$$

with

$$\boldsymbol{v}_j^T(n) = \left[v_j^{(1)}(n), \cdots, v_j^{(k-1)}(n), v_j^{(k+1)}(n), v_j^{(K)}(n)\right] \tag{26}$$

being the LS estimate of all *projected* users at chip time n. Furthermore, denoting the ith *row* of $\mathcal{A}_u$ by $\boldsymbol{a}(i)$

$$\boldsymbol{\Phi}(n) = \sum_{i=1}^{n} \boldsymbol{a}(i)\boldsymbol{a}^T(i) \tag{27}$$

is the $L(K-1) \times L(K-1)$ correlation matrix of the spreading sequences, and

$$\boldsymbol{\theta}(n) = \sum_{i=1}^{n} \tilde{\underline{e}}(i)\boldsymbol{a}(i) \tag{28}$$

is the $L(K-1) \times 1$ crosscorrelation vector between the spreading sequences and the received signal hypothesis.

The standard RLS algorithm for (24) is now easily summarized by the following steps [29, p. 485].

Step 1: Initialize the $L(K-1) \times L(K-1)$-matrix $\mathcal{P}(0) = \delta^{-1}\mathcal{I}$, where δ is a small positive constant, and $\boldsymbol{v}(0) = 0$.

Step 2: $\boldsymbol{\pi}^T(n) = \boldsymbol{a}^T(n)\mathcal{P}(n-1)$, where $\boldsymbol{\pi}^T(n)$ is a $1 \times L(K-1)$ vector.

Step 3:

$$\boldsymbol{k}(n) = \frac{\mathcal{P}(n-1)\boldsymbol{a}(n)}{1+\boldsymbol{\pi}^T(n)\boldsymbol{a}(n)}$$

where $\boldsymbol{k}(n)$ is an $L(K-1) \times 1$ vector, called the *Kalman gain vector*.

Step 4: $\underline{\alpha}(n) = \underline{\tilde{e}}(n) - \boldsymbol{v}^T(n-1)\boldsymbol{a}(n)$, where $\underline{\alpha}$ is a vector of size NL, and its nth entry is the *a priori* error at chip time n.

Step 5: $\boldsymbol{v}(n) = \boldsymbol{v}(n-1) + \boldsymbol{k}(n)\,\underline{\alpha}^T(n)$.

Step 6: $\underline{\beta}(n) = \underline{\tilde{e}}(n) - \boldsymbol{v}^T(n)\boldsymbol{a}(n)$, where $\underline{\beta}(n)$ is the *a posteriori* error at chip time n.

Step 7: $\mathcal{P}(n) = \mathcal{P}(n-1) - \boldsymbol{k}(n)\boldsymbol{\pi}^T(n)$.

Step 8: $\underline{\epsilon}(n) = \underline{\epsilon}(n-1) + \underline{\alpha}^T(n)\,\underline{\beta}(n)$, which is the updated scalar error at chip time n.

Steps 2–8 are executed recursively from $n = 1$ to NL. Clearly, for large L, this turns into a computationally infeasible task since the algorithm needs to be executed for each sequence $\underline{d}_c$. Furthermore, the above brute-force implementation estimates $L(K-1)$ parameters via an RLS algorithm, requiring a complexity of $\mathcal{O}(L^2(K-1)^2)$ per chip, or $\mathcal{O}(NL^2(K-1)^2)$ per symbol and per sequence. In the next subsection we will show that the complexity of this algorithm can be considerably reduced by realizing that it can be executed considering a data window of width only K. This allows us to compute the metric in (22) recursively.

B. Recursive Adaptive Metric Generation

In this subsection we will show that at chip time n, only subvectors of size $1 \times (K-1)$ and submatrices of size $(K-1) \times (K-1)$ are needed in the above algorithm. We will extend the regular RLS algorithm such that new parameters are picked up as the algorithm progresses through the data set, and old parameters are dropped when they are no longer required for the error update. The width of the window of active parameters turns out to be $(K-1)$, the number of interfering users.

Lemma: The calculation of the updated error $\epsilon(n)$ requires only the $K-1$ most recent entries $j \in \{s-(K-1), \cdots, s\}$ of all quantities involved.

Proof: We prove this lemma by induction. First, assume that $\mathcal{P}(n-1)$ has the form as illustrated in Fig. 4, i.e., $P_{ij} = 0$ for $i, j \geq s$, $i \neq j$. This is initially true for $\mathcal{P}(0)$, and we assume it is true for $\mathcal{P}(n-1)$. We show later that $\mathcal{P}(n)$ is also of that form.

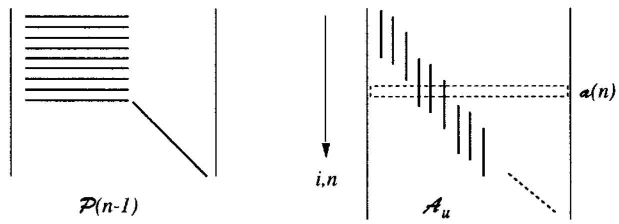

Fig. 4. Illustration of the matrices and vectors used in the extended, recursive RLS algorithm.

The vector $\boldsymbol{a}(n)$ contains at most $K-1$ nonzero elements, corresponding to the at most $K-1$ asynchronous interferers (see Fig. 4). Assume these are the elements from $\max\{1, s-(K-1)\}$ to s.

The update of the *a priori* and *a posteriori* error $\alpha(n)$ and $\beta(n)$ according to Steps 4 and 6 requires only the $K-1$ most recent entries of $\boldsymbol{v}(n-1)$ and $\boldsymbol{v}(n)$ due to $\boldsymbol{a}(n)$. Now $\boldsymbol{\pi}^T(n)$ calculated in Step 2 has zeros in positions $s+1 \rightarrow L(K-1)$ due to the special form of $\mathcal{P}(n-1)$ and $\boldsymbol{a}(n)$. In Step 3 only the $(K-1)$ most recent nonzero coefficients of $\boldsymbol{\pi}^T(n)$ are used. And, likewise in Step 7, since we only require the $(K-1) \times (K-1)$ most recent submatrix of $\mathcal{P}(n)$ (shown below). A similar argument shows that $\boldsymbol{k}(n)$ has zero coefficients in positions $s+1 \rightarrow L(K-1)$ also. Only the $(K-1)$ most recent entries of $\boldsymbol{k}(n)$ are needed in Step 5, since we need only the $(K-1)$ most recent entries of $\boldsymbol{v}(n)$. And also in Step 7, we need only the $(K-1)$ most recent entries of $\boldsymbol{k}(n)$. Since the elements $s+1 \rightarrow L(K-1)$ of both $\boldsymbol{k}(n)$ and $\boldsymbol{\pi}(n)$ equal zero, only the $s \times s$ upper left block of $\mathcal{P}(n-1)$ is updated, and hence $\mathcal{P}(n)$ retains the form assumed and shown in Fig. 4. Furthermore, since we only require the $(K-1) \times (K-1)$ block $i, j \in \{s-(K-1), \cdots, s\}$ of $\mathcal{P}(n-1)$ in the updates of the $(K-1)$ most recent entries of $\boldsymbol{\pi}(n)$ in Step 2, and of $\boldsymbol{k}(n)$ in Step 3, we only need to store and update only the $(K-1) \times (K-1)$ most recent block of $\mathcal{P}(n)$, and indeed of all the vectors involved. □

The advantage of this recursive decomposition is that it allows the calculation of the hypothesis $\underline{\hat{d}}_u$ for sequences $\underline{d}_c$ of arbitrary length, by operating the algorithm in a sliding window mode with window width $K-1$. The complexity of the algorithm is now that of estimating $K-1$ parameters via an RLS algorithm, i.e., it is $\mathcal{O}((K-1)^2)$ per chip, or $\mathcal{O}(N(K-1)^2)$ per symbol.

The recursive adaptive metric generation in the previous section requires that every hypothesized sequence is remembered for proper extension. This leads to a tree search with exponential growth. We reduce the complexity of the decoding algorithm by restarting the algorithm at every new branch. Note that this procedure is optimal only in the synchronous case. The simulations below demonstrate, however, that this method works well also for the asynchronous case.

VI. Simulations

In the following simulations we used a rate $R = 1/2$ convolutional code with four states, and generator polynomials 5, 7 [23]. The results found, however, are very general, as will

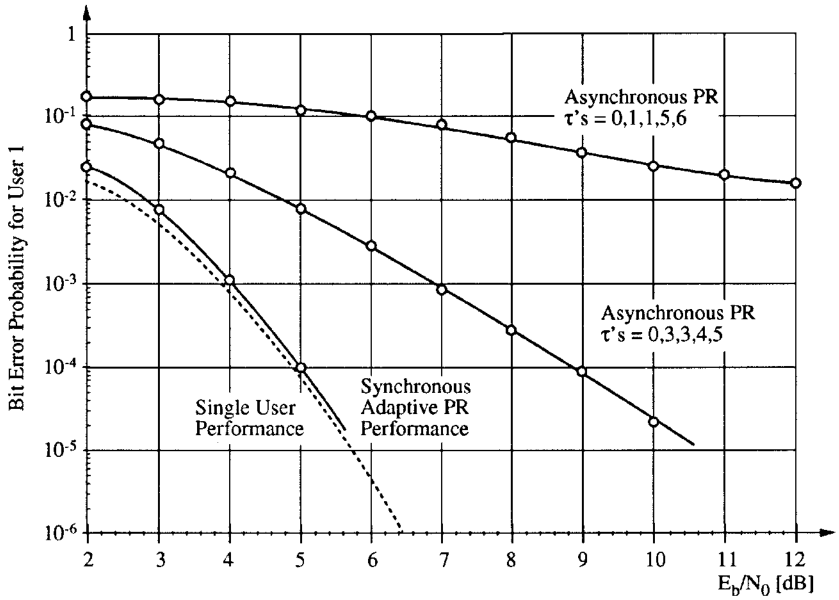

Fig. 5. Simulated performance of the projection receiver for Gold codes of length $N = 7$ for $K = 5$ users.

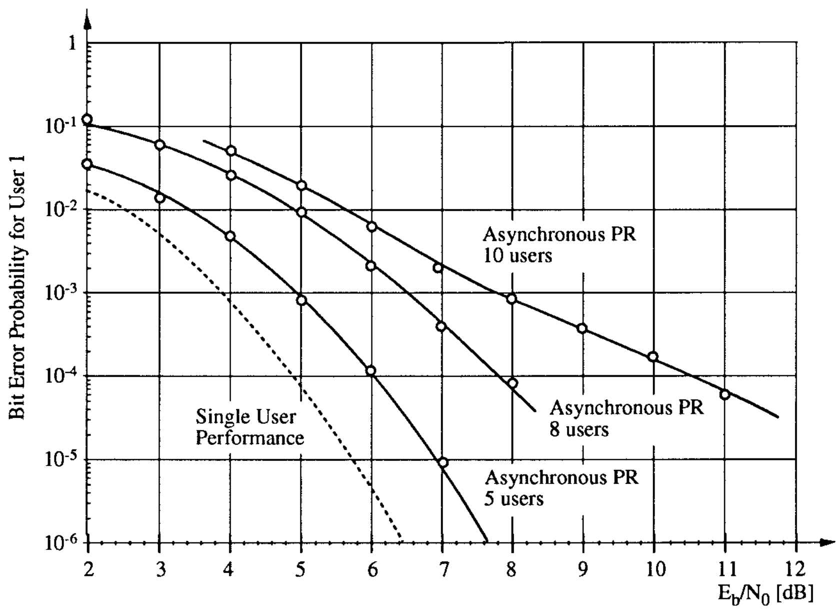

Fig. 6. Simulated performance of the projection receiver for Gold codes of length $N = 15$ for up to $K = 10$ users.

be demonstrated in Section VII, and were further confirmed by simulations with different codes.

Fig. 5 shows the performance of the PR for Gold codes [4] of length $N = 7$ and for $K = 5$ users, giving the following spreading codes (4):

$$\underline{\alpha}_j^{(1)} = +1, -1, +1, +1, +1, -1, -1$$
$$\underline{\alpha}_j^{(2)} = +1, +1, -1, +1, -1, -1, +1$$
$$\underline{\alpha}_j^{(3)} = -1, +1, +1, -1, +1, -1, +1$$
$$\underline{\alpha}_j^{(4)} = -1, +1, -1, +1, -1, -1, -1$$
$$\underline{\alpha}_j^{(5)} = +1, +1, -1, -1, +1, +1, -1.$$

The synchronous PR achieves virtual single-user performance. This is not, however, very surprising, since synchronous Gold codes have excellent crosscorrelation properties. These properties are lost once the users transmit asynchronously, degradations in the order of $\mathcal{L}$ are observable. Combinations of delays can be found (e.g., $\tau^{(1)}=0$, $\tau^{(2)}=1$, $\tau^{(3)}=1$, $\tau^{(4)}=5$, $\tau^{(5)} = 6$) which cause catastrophic error propagation. The reason for this is the existence of combinations like the following: Let the previous bits of users 2 and 5 equal 1 ($d_{j-1}^{(2)} = d_{j-1}^{(5)} = 1$), that of user 3 equal -1 ($d_{j-1}^{(3)} = -1$), and let the present bits of users 2, 3, and 5 be 1 ($d_j^{(2)} = d_j^{(3)} = d_j^{(5)} = 1$). The sum of the signals from users 2, 3, and 5 equals the signal of user 1, and the PR will null the metric at time j for user 1.

Fig. 6 shows the performance of the PR for Gold codes of length 15 for up to ten users. The relative delays of the

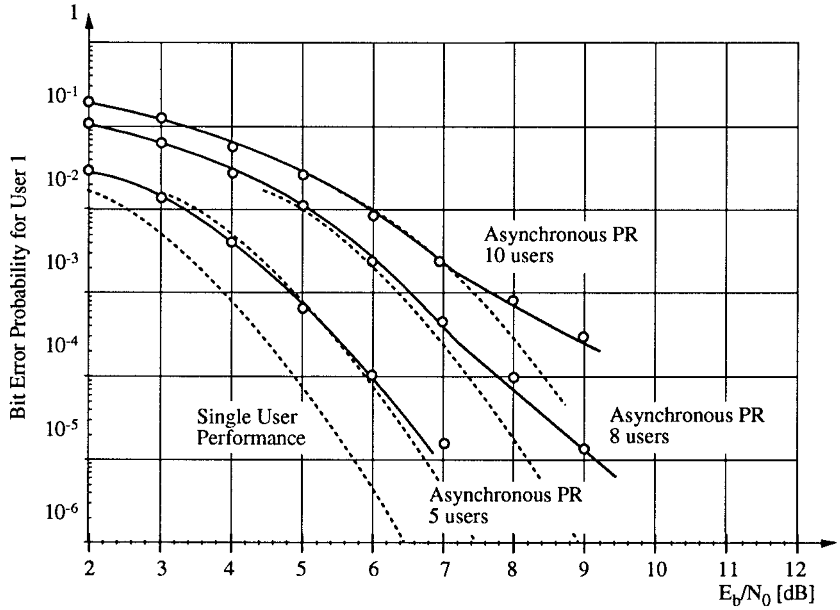

Fig. 7. Simulated performance of the projection receiver for random codes of length $N = 15$ with $K = 4, 6$ and $K = 8$ users with delays $\tau^{(1)} = 0$, $\tau^{(2)} = 3$, $\tau^{(3)} = 5$, $\tau^{(4)} = 6$, $\tau^{(5)} = 7$, $\tau^{(6)} = 9$, $\tau^{(7)} = 11, \tau^{(8)} = 12$.

different users are $\tau^{(1)} = 0$, $\tau^{(2)} = 2$, $\tau^{(3)} = 2$, $\tau^{(4)} = 3$, $\tau^{(5)} = 4$, $\tau^{(6)} = 6$, $\tau^{(7)} = 9$, $\tau^{(8)} = 10$, $\tau^{(9)} = 12$, $\tau^{(10)} = 13$. The performance depends only very little on the relative delays, with the exception of catastrophic delay combinations, and follows well the predicted loss $\mathcal{L}$. For large system loads, e.g., $K = 10$ in Fig. 6, the simulated performance departs from the theoretical performance at low bit-error rates. This is due to inaccuracies of the RLS estimate for high system loads (large number of parameter estimates).

Fig. 7 shows the performance of random spreading sequences of length $N = 15$ for $K = 4, 6$, and $K = 8$ users. The theoretical degradations of 0.97, 2.73, and 3.98 dB are achieved with high accuracy. No dependence of the bit-error rates on the delays could be observed.

Fig. 8 illustrates the practical near–far resistance of the adaptive PR. We plot the degradation suffered with respect to equal power interference, if user 1 is operated at $E_b/N_0 = 4$, 5, 6, and 7 dB for the $N = 15$ asynchronous Gold code system with $K = 8$ (see Fig. 6). The excess power of the interfering users is the additional power of users 2–8 over E_b/N_0 for user 1 (labeled in the figure). Note that the algebraic projection receiver is ideally near–far resistant (no degradation), the loss in the adaptive receiver is due to the adaptive evaluation of the metric.

There is a difference in performance of the fixed CDMA system versus the random CDMA system. While the error rates of the latter follow predictably the theory, the error rates of the former may deviate significantly (Fig. 5) from (1), depending on the geometry of the particular signal set used. The performance of a fixed system can be accurately calculated using (14).

VII. Performance Analysis

In this section we shall study the performance of the asynchronous PR in terms of decoded bit-error probability

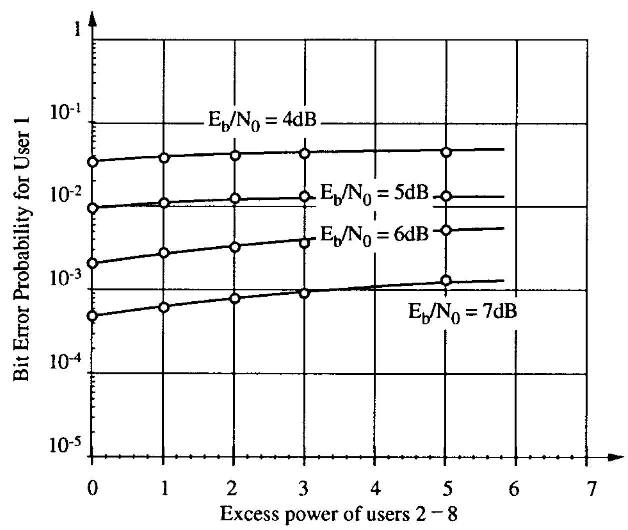

Fig. 8. Simulated near–far resistance of the PR demonstrated for the $N = 15$ Gold code system with $K = 8$ users and asynchronous transmission.

for random spreading codes. Specifically, the degradation of the PR system relative to the interference-free single-user performance will be derived. For mathematical convenience we shall assume unit received powers for all users, i.e.,

$$\mathcal{W} = \underline{I}. \tag{29}$$

The branch metrics for the decoders may be developed as a sum of squared codeword bit distances. We consider a (incorrect) path $\underline{\hat{d}}^{(k)}$ through the single-user trellis corresponding to user k. In order to use the developed performance analysis for convolutional codes we must derive an expression for the probability P_w that the correct path ($\underline{d}^{(k)}$ with Hamming weight w has a metric smaller than that of $\underline{\hat{d}}^{(k)}$). The Hamming weight w error vector

$$\underline{\epsilon} = \underline{\hat{d}}^{(k)} - \underline{d}^{(k)} \tag{30}$$

has w elements equal to ± 2. We place the symbol numbers of these errors into the set $\mathcal{G}$.

The error probability for such an error sequence based on PR metrics of (15) is given by

$$\begin{aligned} P_w &= P(\Lambda(\underline{d}^{(k)}) < \Lambda(\hat{\underline{d}}^{(k)})) \\ &= P\big(\big|\mathcal{M}\big(\underline{e} - \mathcal{A}_c\underline{d}^{(k)}\big)\big|^2 < \big|\mathcal{M}\big(\underline{e} - \mathcal{A}_c\hat{\underline{d}}^{(k)}\big)\big|^2\big) \\ &= P\big(\big|\mathcal{M}\mathcal{A}_c\underline{d}^{(k)}\big|^2 - \big|\mathcal{M}\mathcal{A}_c\hat{\underline{d}}^{(k)}\big|^2 \\ &\quad - 2\underline{e}^T\mathcal{M}\mathcal{A}_c\big(d^{(k)} - \hat{\underline{d}}^{(k)}\big) > 0\big). \end{aligned} \tag{31}$$

Using $\underline{e}^T = \underline{d}^T\mathcal{A} + \underline{n}^T$, and using (29) we obtain

$$\begin{aligned} P_w &= P\big(\big|\mathcal{M}\mathcal{A}_c\underline{d}^{(k)}\big|^2 - \big|\mathcal{M}\mathcal{A}_c\hat{\underline{d}}^{(k)}\big|^2 \\ &\quad - 2\underline{d}^T\mathcal{A}^T\mathcal{M}\mathcal{A}_c\underline{\epsilon} - 2\underline{n}^T\mathcal{M}\mathcal{A}_c\underline{\epsilon} > 0\big) \\ &= P\big(-|\mathcal{M}\mathcal{A}_c\underline{\epsilon}|^2 + 2\underline{n}^T\mathcal{M}\mathcal{A}_c\underline{\epsilon} > 0\big) \\ &= P(\eta_w > 0). \end{aligned} \tag{32}$$

Note that in the step from the first to the second equation above we used

$$\begin{aligned} \underline{d}^T\mathcal{A}^T\mathcal{M}\mathcal{A}_c\underline{\epsilon} &= \underline{d}^{(k)T}\mathcal{A}_c^T\mathcal{M}\mathcal{A}_c\underline{\epsilon} + \underline{d}_u^T\mathcal{A}_u^T\mathcal{M}\mathcal{A}_c\underline{\epsilon} \\ &= \underline{d}^{(k)T}\mathcal{A}_c^T\mathcal{M}\mathcal{A}_c\underline{\epsilon}, \end{aligned} \tag{33}$$

where the second term disappears due to the projection operation, i.e., $\mathcal{A}_u^T\mathcal{M} = 0$.

The random variable η_w is Gaussian-distributed as follows:

$$\eta_w \sim N(-|\mathcal{M}\mathcal{A}_c\underline{\epsilon}|^2, 2N_0|\mathcal{M}\mathcal{A}_c\underline{\epsilon}|^2)$$

which leads to the probability

$$P_w = Q\left(\sqrt{\frac{|\mathcal{M}\mathcal{A}_c\underline{\epsilon}|^2}{2N_0}}\right).$$

We now consider the behavior of P_w over the random code channel via its impact of $\mathcal{M}$ and $\mathcal{A}_c$. Consider the structure of the $N \times N$ submatrix M_{ij} of the projection matrix. This matrix is constructed from blocks of N rows of P as determined by i and j, where $\mathcal{P}$ is an orthonormal basis for null $(\mathrm{span}\{\mathcal{A}_u\})$ of dimension $\bar{u} = L(N-K+1)$. The construction is

$$\begin{aligned} \mathcal{M} &= \mathcal{P}\mathcal{P}^T \\ \Rightarrow M_{i,j} &= P_i P_j^T \end{aligned} \tag{34}$$

where P_i is defined as follows:

$$P_i = \begin{bmatrix} \text{row } iN \text{ of } \mathcal{P} \\ \text{row } iN+1 \text{ of } \mathcal{P} \\ \vdots \\ \text{row } iN+N-1 \text{ of } \mathcal{P} \end{bmatrix} \in \Re^{N,\bar{u}}. \tag{35}$$

Since $\mathcal{P}^T\mathcal{P} = I_{\bar{u}}$

$$\begin{aligned} \mathrm{tr}(\mathcal{P}^T\mathcal{P}) &= \bar{u} \\ &= \mathrm{tr}\left(\sum_{i=i}^{L} P_i^T P_i\right) \\ &= \sum_{i=i}^{L} \mathrm{tr}\big(P_i^T P_i\big). \end{aligned} \tag{36}$$

From this statement we conclude that

$$E\big\{\mathrm{tr}\big(P_i^T P_i\big)\big\} = \frac{\bar{u}}{L} = N - K + 1. \tag{37}$$

In the synchronous channel P_i will have only $N - K + 1$ columns that are nonzero. However, in the asynchronous channel the number of nonzero columns will increase. The effect is due to the smearing of energy that occurs in the asynchronous channel (see (38) at the bottom of this page).

We can now proceed by evaluating the expectation over the channel (39) at the top of the following page, where the inequality follows from the convexity of $Q(\sqrt{x})$ in x. The fourth line follows from the third since the spreading codes are independent and identically distributed (i.i.d.) and P_i as a basis of the unconstrained users' signal space is independent of the vectors $\underline{\alpha}_i^{(k)}$ of the constrained user. We note that the probability of an error depends only on the Hamming weight w of the error path of the constrained user, and, furthermore, that the degradation is identical for the synchronous and the asynchronous channel.

$$\begin{aligned} P_w &= Q\left(\sqrt{\frac{2}{N_0}\sum_{i\in\mathcal{G}}\underline{\alpha}_i^{(k)T}M_{i,i}\underline{\alpha}_i^{(k)} + \frac{1}{N_0}\sum_{i\in\mathcal{G}}\sum_{\substack{j\in\mathcal{G}\\(i\neq j)}}\epsilon_i\underline{\alpha}_i^{(k)T}M_{i,j}\epsilon_j\underline{\alpha}_j^{(k)}}\right) \\ &= Q\left(\sqrt{\frac{2}{N_0}\sum_{i\in\mathcal{G}}\underline{\alpha}_i^{(k)T}P_iP_i^T\underline{\alpha}_i^{(k)} + \frac{1}{N_0}\sum_{i\in\mathcal{G}}\sum_{\substack{j\in\mathcal{G}\\(i\neq j)}}\epsilon_i\underline{\alpha}_i^{(k)T}P_iP_j^T\epsilon_j\underline{\alpha}_j^{(k)}}\right) \\ &= Q\left(\sqrt{\frac{2}{N_0}\sum_{i\in\mathcal{G}}\mathrm{tr}\big(P_i^TP_i\big) + \frac{1}{N_0}\sum_{i\in\mathcal{G}}\sum_{\substack{j\in\mathcal{G}\\(i\neq j)}}\mathrm{tr}\big(P_i^T\epsilon_i\underline{\alpha}_i^{(k)}\epsilon_j\underline{\alpha}_j^{(k)T}P_j\big)}\right). \end{aligned} \tag{38}$$

$$
\begin{aligned}
E_{\mathcal{A}}\{P_w\} &= E_{\mathcal{A}}\left\{ Q\left(\sqrt{ \frac{2}{N_0}\sum_{i\in\mathcal{G}} \operatorname{tr}\left(P_i^T P_i\right) + \frac{1}{N_0}\sum_{i\in\mathcal{G}} \sum_{\substack{j\in\mathcal{G}\\(i\neq j)}} \operatorname{tr}\left(P_i^T \epsilon_i \underline{\alpha}_i^{(k)} \epsilon_j \underline{\alpha}_j^{(k)T} P_j\right) } \right) \right\} \\
&\geq Q\left(\sqrt{ \frac{2}{N_0} E_{\mathcal{A}}\left\{ \sum_{i\in\mathcal{G}} \operatorname{tr}\left(P_i^T P_i\right) \right\} + \frac{1}{N_0} E_{\mathcal{A}}\left\{ \sum_{i\in\mathcal{G}} \sum_{\substack{j\in\mathcal{G}\\(i\neq j)}} \operatorname{tr}\left(P_i^T \epsilon_i \underline{\alpha}_i^{(k)} \epsilon_j \underline{\alpha}_j^{(k)T} P_j\right) \right\} } \right) \\
&= Q\left(\sqrt{ \frac{2}{N_0}\sum_{i\in\mathcal{G}} \frac{\bar{u}}{L} + \frac{1}{N_0}\sum_{i\in\mathcal{G}} \sum_{\substack{j\in\mathcal{G}\\(i\neq j)}} E_{\mathcal{A}}\left\{ \operatorname{tr}\left(P_i^T \epsilon_i \underline{\alpha}_i^{(k)} \epsilon_j \underline{\alpha}_j^{(k)T} P_j\right) \right\} } \right) \\
&= Q\left(\sqrt{ \frac{2}{N_0}\sum_{i\in\mathcal{G}} \frac{\bar{u}}{L} } \right) \\
&= Q\left(\sqrt{ \frac{2w(N-K+1)}{N_0} } \right)
\end{aligned}
\tag{39}
$$

From (39) we can glean a general lower bound for the degradation any coded system suffers. The loss in decibels from the single-user bound using the PR metric of (15) with synchronous or asynchronous random spreading codes is

$$\mathcal{L} \geq 10\log_{10}\left(\frac{N}{N-K+1}\right) \quad \text{dB}. \tag{40}$$

This lower bound on performance loss is very tight for synchronous CDMA (see Figs. 5 and 6). However, in the asynchronous case there is some degradation w.r.t. the bound. This may have several reasons. First, the adaptive decoder operates on a single sample per chip and the convergence of the algorithm is compromised for large relative delays between the users. Second, no extended sequence decoding is used in the asynchronous case, and third, it is unknown whether the asynchronicity causes inherent degradation to the system.

VIII. Concluding Remarks

We have presented a novel projection multiuser receiver which is near–far resistant and has an elegant adaptive implementation for both asynchronous and synchronous CDMA. The performance of our detector as well as its theoretical limits have been demonstrated by theory and simulations for the fully projected receiver where only a single user is decoded via a code trellis. A lower bound on the performance of this detector which is expressed as a power loss factor w.r.t an interference-free system has been derived and shown to be tight. The presented projection receiver is very general and allows for the decoding of several simultaneous users. This topic is left for future investigations.

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