SUBSPACE BLIND DETECTION OF ASYNCHRONOUS CDMA SIGNALS IN MULTIPATH CHANNELS*

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ABSTRACT

This paper presents a blind adaptive detector for asynchronous DS-CDMA in multipath channels. The receiver exploits the underlying subspace structure for recursive estimation of the desired signal vector, and eliminates the need of explicit channel or delay estimation. The detector is constrained to the signal subspace (hence reduced rank), leading to computational efficiency while also benefiting adaptive algorithm performance. It is also shown to be robust to over-estimation of the signal subspace dimension.

1. INTRODUCTION

User asynchronism and multipath contribute to Multi-User Interference (MUI), which is the primary impairment in high rate Code Division Multiple Access (CDMA) systems. Accordingly, joint (multipath) channel estimation along with effective MUI suppression is necessary to guarantee transmission quality for high data rates. Further, a preferable receiver architecture is one that is adaptive and blind (i.e., can operate without training sequences) for greater efficiency.

Several recent works have considered the mitigation of multipath channels. The detector in [1] employs no channel or desired signal estimation and while very attractive in terms of computation, it does not perform satisfactorily in general, since only part of the desired signal energy is utilized. Subspace based receiver architectures that exploit the underlying structure of Direct Sequence CDMA (DS-CDMA) modulation for channel estimation have been proposed by several authors [2, 3, 4, 5] and are accepted as providing avenues for higher performance, at the cost of increased complexity. Our goal in this work is to achieve a balance between these two conflicting ends. Also, we distinguish ourselves from [4] which assumes that the channel length to a negligible fraction of the bit duration (i.e. negligible ISI scenario) and is therefor not applicable for longer channels/higher data rate cases. The receiver in [5] uses a *batch* processing method for channel estimation and subsequent detection, while we emphasize a fully adaptive receiver. We also do not propose direct estimation of the user delays in the receiver, as done by [2].

In [6] we presented a blind adaptive detector with direct desired signal estimation for the case of *synchronous* CDMA in multipath channels. By incorporating the unknown users' delays as part of the the (equivalent) transmission channel, we evaluate the detector performance for the *asynchronous* case. Applying a bank of parallel desired signal estimators, we achieve blind and implicit synchronization, which eliminates the complexity of explicit delay estimation in [2].

2. SYSTEM MODEL

In a K-user asynchronous DS-CDMA system, the transmitted baseband signal of the kth user is

$$x_k(t) = \sqrt{E_k} \sum_{n=-\infty}^{\infty} b_k(n) s_k(t - nT - \tau_k) \qquad (1)$$

where T is the bit interval, τ_k is the transmission delay ($0 \leq \tau_k < T$, assuming frame synchronism), E_k is the transmitted bit energy, and $b_k(n) \in \{-1, 1\}$ is the nth information bit of the kth user. The energy normalized spreading waveform is given by

$$s_k(t) = \sum_{i=0}^{N-1} c_k(i)\psi(t - iT_c) \qquad (2)$$

where c_k ($c_k(i) \in \{-1, 1\}$) is the spreading sequence of the kth user, and $\psi(t)$ is the chip waveform (assumed to rectangular of duration T_c, where $N = T/T_c$ is the processing gain). Different symbols of the same user, as well as symbol sequences of different users, are assumed uncorrelated.

The signal $x_k(t)$ traverses through a multipath channel, resulting in the received signal contributed by user k:

$$r_k(t) = \sum_l \alpha_{k,l} x_k(t - \tau_{k,l}) \qquad (3)$$

where $\alpha_{k,l}$ is the complex path gain of the l-th path, and $\tau_{k,l}$ is the composite delay of path l composed of the transmission delay τ_k and a multipath delay component $\tilde{\tau}_{k,l}$, i.e., $\tau_{k,l} = \tau_k + \tilde{\tau}_{k,l}$. The total received signal is

$$r(t) = \sum_{k=1}^{K} r_k(t) + w(t) \qquad (4)$$

where $w(t)$ is the AWGN. The received signal is sampled at the chip rate after passing through a chip matched filter (the

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receiver clock which is *not* synchronized with the transmitter), leading to the discrete time signal

$$r(n) = \int_{nT_c}^{(n+1)T_c} r(\tau)\psi^*(\tau - nT_c)\, d\tau. \quad (5)$$

For an observation interval of m bits, the received signal vector is

$$\mathbf{r}(n) \stackrel{\text{def}}{=} [r(nN), r(nN+1), \cdots, r((n+m)N-1)]^T. \quad (6)$$

Let the combined response of the chip waveform and the matched filter be $\phi(t) = \psi(t) * \psi(-t)$, and define the shift matrix $\mathbf{T}_l$ so that its elements satisfy

$$t_{i,j} = \begin{cases} 1 & \text{if } j = i - l \\ 0 & \text{otherwise}. \end{cases}$$

The delay of each path can be written as $\tau_{k,l} = (n_{k,l} + \delta_{k,l})T_c$, where $n_{k,l}$ is an integer and $\delta_{k,l} \in [0,1)$. Define $L_k = \max_l \lceil \tau_{k,l}/T_c \rceil$ and $L_{b,k} = \lceil L_k/N \rceil$ denote the order of the effective channel for user k measured in chip and bit intervals, respectively. Furthermore, denote $\mathbf{c}_k$ as the length mN vector so that $\mathbf{c}_k = [c_k(0), \cdots, c_k(N-1), 0, \cdots, 0]^T$. The contribution of path i of user k to the received signal vector is

$$\mathbf{r}_{k,l}(n) = \sum_{i=-L_{b,k}}^{m-1} b_k(n+i)\mathbf{s}_{k,l}^i \quad (7)$$

where the modulating signal vectors are

$$\mathbf{s}_{k,l}^i = \sqrt{E_k}\alpha_{k,l}[\phi(\delta_{k,l})\mathbf{T}_{iN+n_{k,l}}\mathbf{c}_k + \phi(1-\delta_{k,l})\mathbf{T}_{iN+1+n_{k,l}}\mathbf{c}_k]. \quad (8)$$

The received signal vector for user k is

$$\mathbf{r}_k(n) = \sum_l \mathbf{r}_{k,l}(n) = \underbrace{\begin{bmatrix} \mathbf{s}_k^{-L_{b,k}} & \cdots & \mathbf{s}_k^{m-1} \end{bmatrix}}_{\mathbf{S}_k} \underbrace{\begin{bmatrix} b(n-L_{b,k}) \\ \vdots \\ b(n+m-1) \end{bmatrix}}_{\mathbf{b}_k(n)} \quad (9)$$

where the modulating signal vectors are

$$\mathbf{s}_k^i = \sum_l \mathbf{s}_{k,l}^i, \quad i = -L_{b,k}, \cdots, m-1. \quad (10)$$

Clearly $\mathbf{s}_k^i$ is a linear combination of vectors $\mathbf{T}_j\mathbf{c}_k$ for appropriate j. The discrete time counterpart of (4) is thus

$$\mathbf{r}(n) = \sum_{k=1}^{K} \mathbf{r}_k(n) + \mathbf{w}(n) = \underbrace{\begin{bmatrix} \mathbf{S}_1 & \cdots & \mathbf{S}_K \end{bmatrix}}_{\mathbf{S}} \underbrace{\begin{bmatrix} \mathbf{b}_1(n) \\ \vdots \\ \mathbf{b}_K(n) \end{bmatrix}}_{\mathbf{b}(n)} + \mathbf{w}(n). \quad (11)$$

Assuming user 1 is the desired user, we have the equivalent synchronous model [7]

$$\mathbf{r}(n) = \tilde{b}_0(n)\mathbf{u}_0 + \sum_{i=1}^{I} \tilde{b}_i(n)\mathbf{u}_i + \mathbf{w}(n). \quad (12)$$

The desired symbol is $\tilde{b}_0(n) = b_1(n)$, while the interfering symbols $\tilde{b}_i(n), i = 1, \cdots, I$ consist of $m(K-1) + \sum_{k=2}^{K} L_{b,k}$ MUI and $L_{b,1} + m - 1$ ISI symbols for a total interfering dimension of $I = mK + \sum_{k=1}^{K} L_{b,k} - 1$. By defining the code matrix

$$\mathbf{C}_1 = [\mathbf{T}_0\mathbf{c}_1 \;\; \mathbf{T}_1\mathbf{c}_1 \;\; \cdots \;\; \mathbf{T}_{L_1}\mathbf{c}_1], \quad (13)$$

the desired signal vector is

$$\mathbf{u}_0 = \mathbf{s}_1^0 = \mathbf{C}_1\mathbf{h}_1 \quad (14)$$

where $\mathbf{h}_1$ can be viewed as the equivalent channel for user 1. The remaining columns of $\mathbf{S}$ are the interference vectors.

The model derived depends on the desired user's information bits directly and avoids the complications of the model in [2] which is based on *combinations of information bits* that requires receiver synchronization for each path of the desired user [3].

3. SUBSPACE BASED DESIRED SIGNAL ESTIMATION

Construct the data matrix

$$\mathbf{X}(n) = [\mathbf{r}(n) \;\; \mathbf{r}(n-1) \;\; \cdots \;\; \mathbf{r}(n-N_c)] = \mathbf{S}\mathbf{B}(n) + \mathbf{W}(n) \quad (15)$$

where

$$\begin{aligned} \mathbf{B}(n) &= [\mathbf{b}(n) \;\; \mathbf{b}(n-1) \;\; \cdots \;\; \mathbf{b}(n-N_c)], \\ \mathbf{W}(n) &= [\mathbf{w}(n) \;\; \mathbf{w}(n-1) \;\; \cdots \;\; \mathbf{w}(n-N_c)]. \end{aligned} \quad (16)$$

Performing the SVD of $\mathbf{X}^H$ (time index n omitted):

$$\mathbf{X}^H = [\mathbf{U}_s \;\mathbf{U}_n] \begin{bmatrix} \mathbf{\Sigma}_s & \\ & \mathbf{\Sigma}_n \end{bmatrix} \underbrace{\begin{bmatrix} \mathbf{V}_s^H \\ \mathbf{V}_n^H \end{bmatrix}}_{\mathbf{V}^H} \quad (17)$$

reveals the orthogonal signal and noise subspaces $\mathbf{V}_s, \mathbf{V}_n$. We assume $\mathbf{B}(n)$ is of full row rank. Note that $\mathbf{S}$ may be rank deficient, and the true dimension of the signal subspace r_s may be difficult to obtain. Defining $L_b = \max_k L_{b,k}$, we have $r_s = \dim[\text{ran}(\mathbf{S})] \le (L_b + m)K$. In practice an estimate $\hat{r}_s$ is obtained from the SVD and correspondingly, the dimension of $\text{ran}(\mathbf{V}_n)$ is estimated as $\hat{r}_n = mN - \hat{r}_s$. Generally we have $\hat{r}_s \ge r_s$ and therefore

$$\text{ran}(\mathbf{S}) \subseteq \text{ran}(\mathbf{V}_s). \quad (18)$$

It follows by the orthogonality of the signal and noise subspaces that

$$\mathbf{V}_n^H\mathbf{u}_0 = \mathbf{0}. \quad (19)$$

Let $\mathbf{P}_1$ and $\mathbf{P}_1^\perp$ be the orthogonal projection matrices onto $\text{ran}(\mathbf{C}_1)$ and $\text{null}(\mathbf{C}_1^H)$, respectively. From (14) we have

$$\mathbf{P}_1^\perp\mathbf{u}_0 = \mathbf{0}. \quad (20)$$

There are $\hat{r}_n$ and $mN - L_1$ equations in (19), (20), respectively, while the number of unknowns in $\mathbf{u}_0$ is mN. Imposing constraints (19) and (20) simultaneously, a unique (up to an unknown complex gain) solution of $\mathbf{u}_0$ exists if

$$\dim[\text{ran}(\mathbf{V}_s) \cap \text{ran}(\mathbf{C}_1)] = 1. \tag{21}$$

A necessary condition for (21) to hold is that the number of linear equations equals or exceeds the number of unknowns less 1, i.e.,

$$\hat{r}_n \geq L_1 - 1. \tag{22}$$

For $K < N$, condition (22) can always be satisfied by increasing m. Simulation shows (21) is generally true, even if dimension of $\mathbf{V}_s$ is over-estimated, which is indicative of the robustness of the method (i.e., $\mathbf{S}$ is not required to be full rank).

Because in practice we only have estimates of the subspace, (19) and (20) can be solved in the least squares sense

$$\min_{\mathbf{u}_0} \mathbf{u}_0^H(\mathbf{V}_n\mathbf{V}_n^H + \mathbf{P}_1^\perp)\mathbf{u}_0 \quad \text{s.t.} \quad ||\mathbf{u}_0||_2 = 1. \tag{23}$$

The solution is the minimal eigenvector of the matrix $\mathbf{V}_n\mathbf{V}_n^H + \mathbf{P}_1^\perp$. Since $\mathbf{P}_1 + \mathbf{P}_1^H = \mathbf{I}$ and $\mathbf{V}_s\mathbf{V}_s^H + \mathbf{V}_n\mathbf{V}_n^H = \mathbf{I}$, (23) can be transformed into a maximization problem:

$$\max_{\mathbf{u}_0} J = \mathbf{u}_0^H \mathbf{Q}\, \mathbf{u}_0 \quad \text{s.t.} \quad ||\mathbf{u}_0||_2 = 1, \tag{24}$$

whose solution is the maximal eigenvector of matrix $\mathbf{Q} \stackrel{\text{def}}{=} \mathbf{V}_s\mathbf{V}_s^H + \mathbf{P}_1$.

4. BLIND ADAPTIVE DETECTION AND SYNCHRONIZATION

For adaptive subspace estimation/tracking, we adopt the efficient Refinement Only Fast Subspace Tracker (RO-FST) [8]. Thereafter, the simple power method [9, Sec. 8.2] can be used to iteratively solve (24) via

$$\begin{aligned} \mathbf{a} &= \hat{\mathbf{Q}}(n)\hat{\mathbf{u}}_0(n-1), && (25)\\ \hat{\mathbf{u}}_0(n) &= \mathbf{a}/||\mathbf{a}||_2. && (26) \end{aligned}$$

Since the linear MMSE detector is known to lie in the signal subspace [5], a reduced rank adaptive detector constrained to the signal subspace can benefit from faster convergence in situations where the smallest signal eigenvalue is significantly greater than the noise eigenvalues. Rank reduction also means reduced detector complexity and is natural in this context, since the signal subspace is already extracted. Using the signal subspace extracted by (17), the reduced rank detector $\mathbf{d}(n)$ is obtained by solving

$$\hat{\mathbf{V}}_s(n)\hat{\mathbf{\Sigma}}_s^H(n)\hat{\mathbf{\Sigma}}_s(n)\hat{\mathbf{V}}_s^H(n)\mathbf{d}(n) = \hat{\mathbf{u}}_0(n), \tag{27}$$

which can be done efficiently since the columns of $\hat{\mathbf{V}}_s(n)$ are orthonormal and triangular $\hat{\mathbf{\Sigma}}_s$ is produced by RO-FST.

The method described above is based on modeling the transmission delay as part of the equivalent channel $\mathbf{h}_1$ which effectively increases its order. While this avoids the complexity of explicit delay estimation in [2], it also introduces additional and possibly redundant detector coefficients. If the transmission delay was known (or estimated such as by [2]), the length of $\mathbf{h}_1$ can be reduced to only the multipath delay spread, thereby improve performance of the adaptive algorithm. Indeed the fewer columns $\mathbf{C}_1$ has, the better performance the detector achieves. We assume that a rough estimate of the desired user's multipath delay spread (which can be an over-estimation) is available, denoted as L_m chip durations. When the transmission delay is unknown, we can apply a bank of M ($1 \leq M \leq N$) desired signal estimators, with assumed channel order

$$L_e = \lceil \frac{N}{M} \rceil + L_m - 1. \tag{28}$$

Note that $L = N + L_m - 1$ is the maximal possible channel order for the desired user. Let the code matrices in (13) for each estimator be

$$\begin{aligned} \mathbf{C}_{1,i} = {} & [\mathbf{T}_{(i-1)\Delta}\mathbf{c}_1 \quad \mathbf{T}_{(i-1)\Delta+1}\mathbf{c}_1 \quad \cdots \quad \mathbf{T}_{(i-1)\Delta+L_e}\mathbf{c}_1],\\ & i = 1, \cdots, M, \end{aligned} \tag{29}$$

where

$$\Delta = L_e - L_m + 1. \tag{30}$$

Such parameters guarantee that at least one code matrix corresponds to the true delays of the desired user, while keeping L_e as small as possible. The best branch is chosen by

$$\max_i J_i = \hat{\mathbf{u}}_{0,i}^H \hat{\mathbf{Q}}_i \hat{\mathbf{u}}_{0,i} \tag{31}$$

where $\hat{\mathbf{u}}_{0,i}$ and $\hat{\mathbf{Q}}_i$ are the corresponding estimates for branch i. In this way, the nominal order of channel $\mathbf{h}_1$ is reduced from L to L_e, and the performance of the adaptive detector is improved at the expense of some increase in receiver complexity. The initial synchronization is realized blindly and implicitly, after which fine tuning can be performed.

5. SIMULATION RESULTS

Two examples are shown here, with 4 interfering users, each with MUI $= 20\,$dB (the desired user's energy E_1 is normalized to $0\,$dB). The processing gain is $N = 16$ and the received bit SNR is 20 dB. The spreading codes and transmission delays were randomly generated (delays uniform over a bit duration), and kept fixed over all runs. The multipath channels were generated with Jakes model with the fading rate set to 0 to mimic essentially static multipath. Fig. 1 depicts the channel response for the desired user. The smoothing factor was set to $m = 2$, and the forgetting factor in RO-FST $\beta = 0.997$. The averaged output Signal-to-Interference-and-Noise Ratio at the nth iteration was computed as

$$\text{SINR}(n) = \frac{\sum_{i=1}^{M_c} |\mathbf{d}_i^H(n)\mathbf{u}_0|^2}{\sum_{i=1}^{M_c} |\mathbf{d}_i^H(n)[\mathbf{r}_i(n) - b_1(n)\mathbf{u}_0]|^2} \tag{32}$$

and the results shown are based on averages over $M_c = 200$ Monte Carlo runs. The ideal output SINR of the MMSE detector is calculated from [10]:

$$\text{SINR}_* = E_1|\mathbf{u}_0^H(\mathbf{R} - E_1\mathbf{u}_0\mathbf{u}_0^H)^{-1}\mathbf{u}_0|. \tag{33}$$

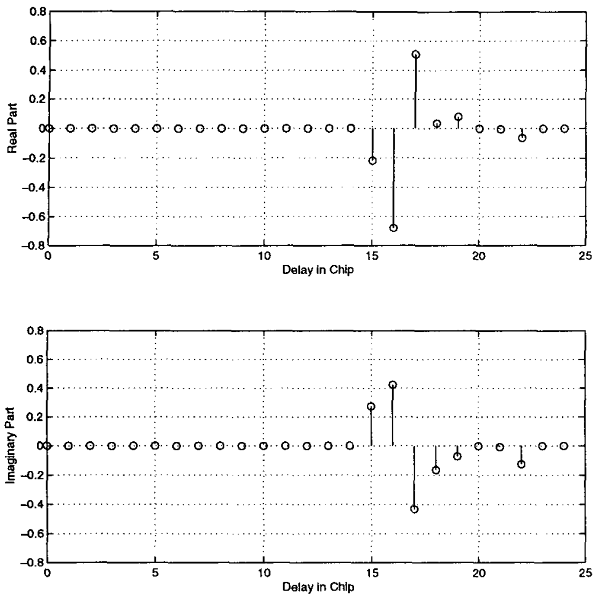

Figure 1. Equivalent Channel Impulse Response of the Desired User.

We set the over-estimated multipath delay spread of the desired user to be $L_m = 9$. The numbers of parallel desired signal estimators are $M = 3$ and 6, with the assumed channel order $L_e = 14$ and 11, respectively. Figs. 2 and 3 show the detector performance. For comparison, the ideal output SINR=14.9 dB is shown — we note that the output SINR reaches close to optimal performance typically within 200 symbols, and performance improves with bigger M at the cost of more complexity.

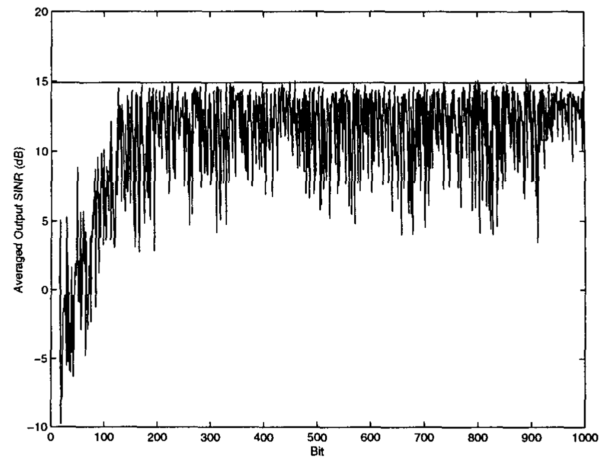

Figure 2. Performance of the detector, $N = 16, K = 5, m = 2, M = 3, L_e = 14$, bit SNR $= 20$ dB.

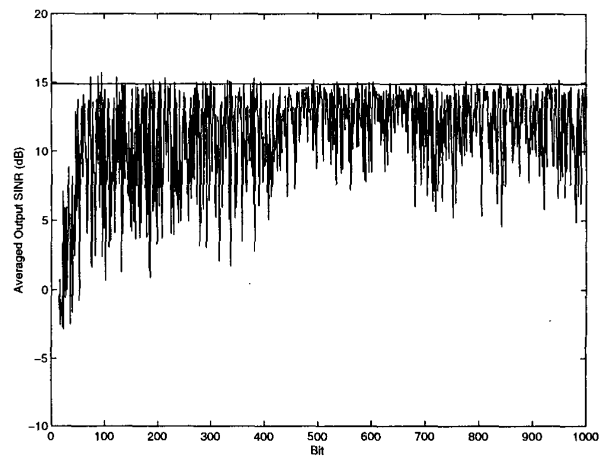

Figure 3. Performance of the detector, $N = 16, K = 5, m = 2, M = 6, L_e = 11$, bit SNR $= 20$ dB.

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