Subspace Based Blind Detector for Multi-Carrier CDMA with Virtual Carriers over Dispersive Channels

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Abstract

Blind MMSE detection for Multi-Carrier CDMA (MC-CDMA) [1] systems in the presence of Intersymbol Interference (ISI) channels is addressed. By exploiting the presence of virtual carriers *[2], a blind detector architecture - (subspace based) channel estimator followed by a linear MMSE equalizer - is proposed that achieves high spectral efficiency due to elimination of cyclic prefix while not requiring any training for start-up. Both full and reduced dimension subspace detection is investigated, to highlight possible performance-complexity trade-offs suitable for uplink and downlink applications, respectively.*

1 Introduction

Interest in Orthogonal Frequency-Division Multiplexing [3][4] (OFDM) has witnessed a rebirth in the context of next generation high-speed wireless/mobile communications systems due to its many advantages - notably, its high spectral efficiency, frequency diversity to combat frequency selective fading, and design efficiency as well. Hybrids that combine OFDM with Code Division Multiple Access (CDMA) have been proposed to equip an OFDM with multiple access capability. These have led to Multi-Carrier CDMA and other variants (see [5] for a good survey). All these hybrid schemes inherent the spirit of OFDM - parallel transmission over large number of narrow band orthogonal carriers to give greater immunity against dispersive channels as compared to DS-CDMA for the same transmitted bandwidth.

We focus on the dispersive channel, which results in ISI and thus arises the primary impairment for high data-rate systems. Reliable channel estimation is required for the coherent receiver adopted here. The traditional approach to channel estimation uses a cyclic prefix of length greater than the delay spread of the channel to absorb the ISI [5]; this converts the linear convolution in time (due to the channel) to a (complex) scaling factor in frequency on each sub channel. Thus, a one-tap frequency domain equalizer suffices to compensate for the sub channel distortion, however, at the cost of $10\% - 25\%$ spectral efficiency loss [4] due to the cyclic prefix. The sub-channel coefficients are estimated by the insertion of (periodic) training symbols in the transmitted data sequence, incurring a loss in channel utilization.

Our work is motivated by systems with high spectral efficiency which naturally leads to design approaches with no cylic prefix and *blind channel estimation* methods. In this paper, we demonstrate how the presence of 'virtual carriers' may be exploited for this purpose. The resulting blind detector architecture consists of a subspace based channel estimator [6] followed by a linear MMSE detector. Upon decomposition of the observation space into mutually orthogonal signal and noise subspaces, the channel information is estimated using the knowledge of the desired user's spreading code as in [7]; the channel estimate is then used to construct linear MMSE detector via direct off-line computation.

The rest of the paper is organized as follows: a baseband signal model for MC-CDMA system is introduced in Section 2. The subspace based channel estimator (both full and reduced dimension versions) and the linear MMSE detector are developed respectively in Section 3. Computer simulations are conducted in Section 4 to show the effectiveness of the proposed detector. Section 5 concludes the paper with final remarks.

2 Baseband Signal Model

Consider an MC-CDMA system (see Fig. 1) with Q sub-carriers of which only P are modulated by user's data symbols. The remaining $Q - P$ carriers are unmodulated to avoid transmit filtering and constitute *virtual carriers*. *No* cyclic prefix is used. K users are multiplexed

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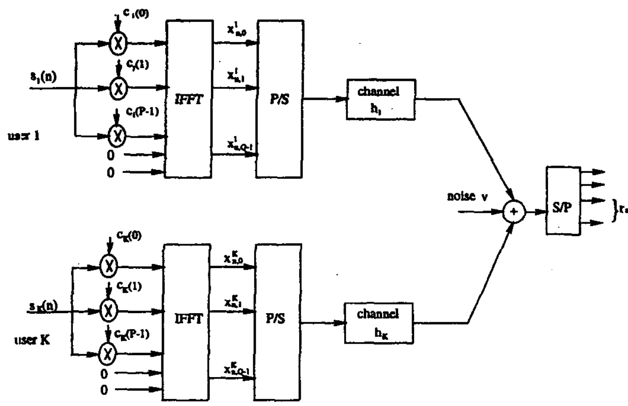

Figure 1. Baseband MC-CDMA system model

via their distinct P-length spreading sequences ($\mathbf{c}_i = \begin{bmatrix} c_i(0) & c_i(1) & \ldots & c_i(P-1) \end{bmatrix}^T$ for the i-th user). Denoting the transmitted information symbol for user i in duration n by $s_i(n)$, the signal after the IFFT operation is

$$\mathbf{x}_{i,n} = [x_{i,n}(0), \ldots, x_{i,n}(Q-1)]^T = \mathbf{W}_P\, \mathbf{c}_i\, s_i(n), \quad (1)$$

where $\mathbf{W}_P$ is the $Q \times P$-dimensional partial IDFT matrix

$$\mathbf{W}_P = \frac{1}{Q}\begin{bmatrix} 1 & 1 & \ldots & 1 \\ 1 & W_Q^{-1} & \ldots & W_Q^{-(P-1)} \\ 1 & W_Q^{-2} & \ldots & W_Q^{-2(P-1)} \\ \vdots & & & \vdots \\ 1 & W_Q^{-(Q-1)} & \ldots & W_Q^{-(Q-1)(P-1)} \end{bmatrix}$$

with $W_Q = e^{-j2\pi/Q}$. The dispersive channels are assumed to have length $L+1$ and $Q - L > P$. The channel impulse response is denoted by $\mathbf{h}_i = [h_i(0), \ldots, h_i(L)]^T$ for user i. For convenience, the i-th user's transmit amplitude $\sqrt{E_i}$ is lumped into $\mathbf{h}_i$.

Upon transmission of $\mathbf{x}_{i,n}$'s through the channel, the output is collected in a Q-vector

$$\bar{\mathbf{r}}_n = [r_n(0), \ldots, r_n(Q-1)]^T = \sum_{i=1}^{K} \mathbf{x}_{i,n} * \mathbf{h}_i + \bar{\mathbf{v}}_n \quad (2)$$

where $*$ respresents linear convolution, $\bar{\mathbf{v}}_n$ is an additive White Gaussian Noise (AWGN) vector consisting of independent, zero mean and variance N_0 components. The presence of the channel causes ISI from the past symbol in $\mathbf{r}_n$; thus we remove the first L ISI affected samples and collect the remaining (ISI free) samples ($r_n(L)$ to $r_n(Q-1)$) in duration n into a vector

$$\mathbf{r}_n = \sum_{i=1}^{K} \mathbf{H}_i \mathbf{W}_P \mathbf{c}_i s_i(n) + \mathbf{v}_n = \mathbf{A}\mathbf{s}_n + \mathbf{v}_n. \quad (3)$$

with $\mathbf{s}_n = [s_1(n), \ldots, s_K(n)]^T$ and $\mathbf{A} = [\mathbf{H}_1\mathbf{W}_P\mathbf{c}_1, \ldots, \mathbf{H}_K\mathbf{W}_P\mathbf{c}_K]$.

Introducing the time-reversed version of $\mathbf{h}_i$ as $\bar{\mathbf{h}}_i = [h_i(L), \ldots, h_i(0)]^T$, the $Q - L \times Q$-dimensional Toeplitz matrix $\mathbf{H}_i$ is given by

$$\begin{aligned} \mathbf{H}_i &= Toeplitz(\bar{\mathbf{h}}_i) \\ &= \begin{bmatrix} h_i(L) & \ldots & h_i(0) & & & \\ & h_i(L) & \ldots & h_i(0) & & \\ & & \ddots & & \ddots & \\ & & & h_i(L) & \ldots & h_i(0) \end{bmatrix} \end{aligned}$$

Further, define (borrowing notations from $MATLAB^{TM}$):

$$\begin{aligned} \bar{\mathbf{W}} &= \mathbf{W}_P(1:Q-L, 1:P), & (4) \\ \mathbf{W}_L &= \mathbf{W}_P(1:P, 1:L+1), & (5) \end{aligned}$$

where the ranges in parentheses represent the rows and columns of the corresponding matrix. Then, the scaled (by Q) IFFT output $\bar{\mathbf{g}}_i$, and $\mathbf{g}_i$ constructed from $\bar{\mathbf{g}}_i$'s first P elements are, respectively

$$\begin{aligned} \bar{\mathbf{g}}_i &= [g_i(0), \ldots, g_i(Q-1)]^T = Q\mathbf{W}_P\bar{\mathbf{h}}_i, & (6) \\ \mathbf{g}_i &= [g_i(0) \ldots, g_i(P-1)]^T = Q\mathbf{W}_L\bar{\mathbf{h}}_i. & (7) \end{aligned}$$

Now note that it can be shown the columns of $\mathbf{A}$ have the following structure:

$$\mathbf{H}_i\mathbf{W}_P\mathbf{c}_i = Q\bar{\mathbf{W}}diag(\mathbf{c}_i)\mathbf{W}_L\bar{\mathbf{h}}_i \quad (8)$$

3 Blind MMSE Detection

Without loss of generality, user 1 is denoted as the desired user and blind channel estimation of the channel vector $\mathbf{h}_1$ via a subspace approach is outlined next. For notational convenience, the time index n will be omitted in the following where there is no confusion.

3.1 Subspace Based Blind Detection: Full Dimension

A subspace based channel estimation procedure can be readily derived from the signal model (3) provided that the necessary condition $Q - L \geq K$ for $\mathbf{A}$ to have full column rank is satisfied. The resulting algorithm exploits the special structure of $\mathbf{A}$ in (8) and is motivated by the approach in [7].

Assume each user's transmitted information symbols s_i's to be i.i.d. sequences with zero mean and unity variance. Consider the EigenValue Decomposition (EVD) on the correlation matrix of the observation vector $\mathbf{r}$:

$$\mathbf{R_r} = E[\mathbf{r}\mathbf{r}^H] = \begin{bmatrix} \bar{\mathbf{U}}_s & \bar{\mathbf{U}}_n \end{bmatrix} \begin{bmatrix} \bar{\mathbf{\Lambda}}_s & \\ & \bar{\mathbf{\Lambda}}_n \end{bmatrix} \begin{bmatrix} \bar{\mathbf{U}}_s^H \\ \bar{\mathbf{U}}_n^H \end{bmatrix} \quad (9)$$

where $[\bar{\mathbf{U}}_s, \bar{\mathbf{U}}_n]$ is an $Q-L \times Q-L$ unitary matrix. The K columns of $\bar{\mathbf{U}}_s$ span the signal subspace, while $Q-L-K$ column vectors of $\bar{\mathbf{U}}_n$ span the orthogonal complement to the signal subspace, known as the noise subspace. $\bar{\mathbf{\Lambda}}_s = diag(\lambda_1, \lambda_2, \ldots, \lambda_K)$ is a diagonal matrix consisting of K significant eigenvalues corresponding to the signal subspace, while $\bar{\mathbf{\Lambda}}_n = N_0 I_{Q-L-K}$ are the noise subspace eigenvalues. Since $\mathbf{H}_1 \mathbf{W}_P \mathbf{c}_1$ lies in the subspace spanned by $\bar{\mathbf{U}}_s$, by the orthogonality property between signal subspace and noise subspace, it is orthogonal to $\bar{\mathbf{U}}_n$, namely by (8)

$$\bar{\mathbf{U}}_n^H \mathbf{H}_1 \mathbf{W}_P \mathbf{c}_1 = \bar{\mathbf{U}}_n^H \bar{\mathbf{W}} diag(\mathbf{c}_1) \mathbf{W}_L \bar{\mathbf{h}}_1 = 0 \qquad (10)$$

Eqn. (10) results in $Q-L-K$ equations for $L+1$ unknowns. Combining with the requirement that $Q-L>P$ and the necessary condition for the identifiability (which needs A to be full column rank, i.e., $Q-L \geq K$), the number of users K_f supported by the full-dimensioned system must satisfy

$$K_f < \frac{2Q-3L-1}{2} \qquad (11)$$

For the practical situation when the signal and noise subspaces are estimated from the sample covariance matrix $\hat{\mathbf{R}}_r$, the above suggests the channel estimator

$$\hat{\bar{\mathbf{h}}}_1 = \arg \min_{\|\bar{\mathbf{h}}_1\|=1} \|\hat{\bar{\mathbf{U}}}_n^H \bar{\mathbf{W}} diag(\mathbf{c}_1) \mathbf{W}_L \bar{\mathbf{h}}_1\|^2 \qquad (12)$$

where $\hat{\bar{\mathbf{U}}}_n$ denotes the estimated noise subspace from $\hat{\mathbf{R}}_r$. The solution to the above is well-known to be the eigenvector of $\mathbf{W}_L^H diag(\mathbf{c}_1)^H \bar{\mathbf{W}}^H \hat{\bar{\mathbf{U}}}_n \hat{\bar{\mathbf{U}}}_n^H \bar{\mathbf{W}} diag(\mathbf{c}_1) \mathbf{W}_L$ corresponding to the minimum eigenvalue.

3.2 Reduced-Dimension Blind Channel Estimation

The complexity of the channel estimator presented above is $O((Q-L)^3)$ due to the EVD of the sample covariance matrix of the full-dimensioned received signal model in (3). In several scenarios, this complexity may be prohibitive for practical implementation - such as in downlink applications where the algorithm must be implemented in the transceiver on a mobile handset. Motivated by this, we now derive a reduced-dimension received signal model that leads to an $O(P^3)$ channel estimator with significant complexity reductions when $P << Q-L$.

Assuming that the necessary condition (8) is also sufficient in practice, the left pseudo-inverse matrix of $\bar{\mathbf{W}}$, i.e., $\tilde{\mathbf{W}} = (\bar{\mathbf{W}}^H \bar{\mathbf{W}})^{-1} \bar{\mathbf{W}}^H$ can be applied to (3) yielding

$$\tilde{\mathbf{y}} = \tilde{\mathbf{W}} \mathbf{r} = [\ \mathbf{f}_1 \quad \mathbf{f}_2 \quad \ldots \quad \mathbf{f}_K\ ] \mathbf{s} + \mathbf{z} \qquad (13)$$

where from (8), $\mathbf{f}_i = Q diag(\mathbf{c}_i) \mathbf{W}_L \bar{\mathbf{h}}_i$. Note that the additive noise component $\mathbf{z}$ is now colored with covariance given by $\mathbf{R}_z = N_0 (\bar{\mathbf{W}}^H \bar{\mathbf{W}})^{-1}$. Since $(\bar{\mathbf{W}}^H \bar{\mathbf{W}})^{-1}$ is a positive define *Hermitian* matrix, it admits a Cholesky factorization $(\bar{\mathbf{W}}^H \bar{\mathbf{W}})^{-1} = \mathbf{B}\mathbf{B}^H$. An equivalent received signal model in additive *white* noise results by applying $\mathbf{B}^{-1}$ to the signal vector $\tilde{\mathbf{y}}$ in (13), i.e.,

$$\mathbf{y} = \mathbf{B}^{-1} \tilde{\mathbf{y}} = \mathbf{B}^{-1} \tilde{\mathbf{W}} \mathbf{r} \qquad (14)$$

A low complexity channel estimator based on (14) can be obtained after the raw observation vector $\mathbf{r}$ is prefiltered via $\mathbf{D} = \mathbf{B}^{-1} \tilde{\mathbf{W}}$ to produce the (reduced) P dimension observation vector $\mathbf{y}$. The pre-filtering results in no loss of information for purposes of detection, i.e., the observation $\mathbf{y}$ is a sufficient statistic.

Similar to the full dimension estimator, the covariance matrix of the observation vector $\mathbf{y}$ can be decomposed via EVD as

$$\mathbf{R}_y = E[\mathbf{y}\mathbf{y}^H] = [\ \mathbf{U}_s \quad \mathbf{U}_n\ ] \begin{bmatrix} \mathbf{\Lambda}_s & \\ & \mathbf{\Lambda}_n \end{bmatrix} \begin{bmatrix} \mathbf{U}_s^H \\ \mathbf{U}_n^H \end{bmatrix} \qquad (15)$$

where the signal subspace is spanned by the K columns of $\mathbf{U}_s$, while the noise subspace is now spanned by $P-K$ column vectors of $\mathbf{U}_n$; $\mathbf{\Lambda}_s = diag(\lambda_1, \lambda_2, \ldots, \lambda_K)$ and $\mathbf{\Lambda}_n = N_0 I_{P-K}$ are diagonal matrices constituted by eigenvalues corresponding to the signal subspace and noise subspace respectively.

When only estimated noise and signal subspaces ($\hat{\mathbf{U}}_n$ and $\hat{\mathbf{U}}_s$) are available from EVD of sample covariance matrix $\hat{\mathbf{R}}_y$, the desired user's channel vector $\bar{\mathbf{h}}_1$ is determined by the Least-Square (LS) solution of

$$\hat{\mathbf{U}}_n^H \mathbf{B}^{-1} diag(\mathbf{c}_1) \mathbf{W}_L \bar{\mathbf{h}}_1 \approx 0 \qquad (16)$$

Eqn. (16) results in $P-K$ equations for $L+1$ unknowns. Combining with the requirement that $Q-L>P$ and the necessary condition for the identifiability (which needs $\mathbf{B}^{-1} \tilde{\mathbf{W}} \mathbf{A}$ to be full column rank, i.e., $P > K$), the number of users in the reduced dimension system must satisfy

$$K_r < \frac{Q+P-2L-1}{2} \quad \Rightarrow \quad K_r \leq \frac{Q+P}{2} - L. \qquad (17)$$

3.3 MMSE Detector Computation

Once $\hat{\bar{\mathbf{h}}}_1$ is available, the optimal coefficient vector $\mathbf{w}_{opt}$ for a linear MMSE equalizer for the desired user is determined by solving $\mathbf{w}_{opt} = \arg\min_{\mathbf{w}} E[\|s_1 - \hat{s_1}\|^2]$, where $\hat{s_1}$ is the estimate of the transmitted symbol s_1

$$\hat{s_1} = \mathbf{w}^H \mathbf{r} \ (\textit{full dimension detector}) \qquad (18)$$

$$\hat{s_1} = \mathbf{w}^H \mathbf{y} \ (\textit{reduced dimension detector}) \qquad (19)$$

A. Full Dimension MMSE Detector: The optimal equalizer weight vector for the full dimension detector $\mathbf{w}_{\mathbf{opt},\mathbf{r}}$ which minimizes the MSE between s_1 and its estimate $\hat{s}_1$ is

$$\mathbf{w}_{opt,r} = \bar{\mathbf{U}}_s \bar{\Lambda}_s^{-1} \bar{\mathbf{U}}_s^H \bar{\mathbf{W}} diag(\mathbf{c}_1) \mathbf{W}_L \hat{\mathbf{h}}_1 \quad (20)$$

In practice, $\bar{\mathbf{U}}_s$ and $\bar{\Lambda}_s$ are replaced by their estimated version $\hat{\bar{\mathbf{U}}}_s$ and $\hat{\bar{\Lambda}}_s$.

B. Reduced Dimension MMSE Detector: Similarly, the optimal equalizer weight vector $\mathbf{w}_{\mathbf{opt,y}}$ for the reduced dimension detector is

$$\mathbf{w}_{opt,y} = \hat{\mathbf{U}}_s \hat{\Lambda}_s^{-1} \hat{\mathbf{U}}_s^H \mathbf{B}^{-1} diag(\mathbf{c}_1) \mathbf{W}_L \hat{\mathbf{h}}_1 \quad (21)$$

It is worth remarking that the optimum MMSE detector lies in signal subspace, as is evident from (20) or (21).

4 Simulation Results

Monte Carlo simulations were conducted to assess the effectiveness of the proposed blind detectors. The channels were generated using the method of [8] that models the channel as a finite length, tapped delay line with an input Doppler frequency parameter that controls the rate of time variation. We set the Doppler frequency to zero to get time-invariant ISI channels.

We consider an MC-CDMA system with no cyclic prefix with $Q = 16$. Number of data subcarriers P is set to 10 unless otherwise noted; the channel length $L + 1$ is 3 or 4. P-length random binary codes are applied to BPSK data symbols at the transmitter during each run; the codes and channel realizations are independently selected for each run of the simulation, thereby resulting in averaged performance over the choice of such codes and channels. The i-th user's transmit SNR is defined as $SNR_i \stackrel{\text{def}}{=} 10\log_{10}\frac{E_i}{N_0}$. Both single user and multiple users cases are simulated. The sample average of the autocorrelation matrix $\mathbf{R_r}$ (or $\mathbf{R_y}$) is obtained by using 1000 samples of $\mathbf{r}_n$ (or $\mathbf{y}_n$). All results reported are obtained by averaging over 300 independent Monte Carlo runs.

Performance of the channel estimator: Performance of the proposed channel estimator is evaluated in terms of normalized Root Mean-Square Error (RMSE). Fig. 2 illustrates the RMSE comparison between the full and reduced dimension channel estimator when two active users are present. The channel length $L + 1$ is 3 and 4.

The comparison clearly shows that reduced dimension estimator suffers higher RMSE than the full dimension estimator, with their performance converging at sufficiently high SNR (> 20dB). Further, increasing the channel order by 1 incurs a maximum 3dB penalty for fixed K.

Next, the impact of increasing the number of users is shown in Fig. 3, where the channel length is set to 3 and the SNR is fixed at 15dB. The lower rate of RMSE increasing for the full dimension estimator reflects its better noise tolerance property over the dimension-reduced estimator.

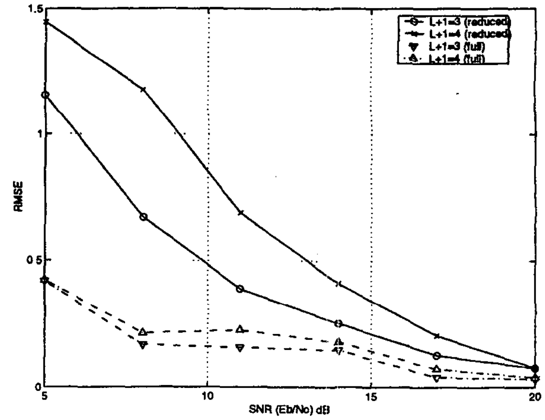

Figure 2. RMSE comparison between full and reduced dimension estimator

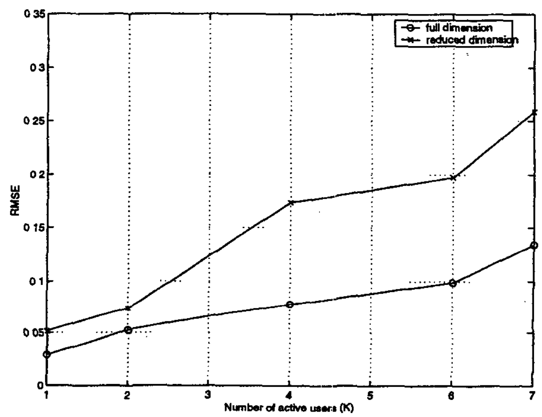

Figure 3. RMSE Vs. number of active users K

It is also interesting to investigate the estimator's robustness to overestimate of the channel length $L + 1$. In practice the channel length is selected based on the maximum channel spread value of the particular application. Thus the chosen value may be an overestimate over the actual length. Fig. 4 shows the RMSE vs. the estimated channel length, where $K = 2$, input SNR is set to $20dB$, and the actual length is 3. As is seen, RMSE gets larger as the estimated channel length drifts away from the true value; also the reduced-dimension estimator incurs higher error than the full-dimension one does, meaning the latter estimator is less sensitive to the channel length overestimation.

BER Performance of the MMSE Detector: Bit Error Rate (BER) is plotted as an indicator of the MMSE detector. In Fig. 5, the BER curves for both the full and reduced dimension detector are shown together for comparison, where the

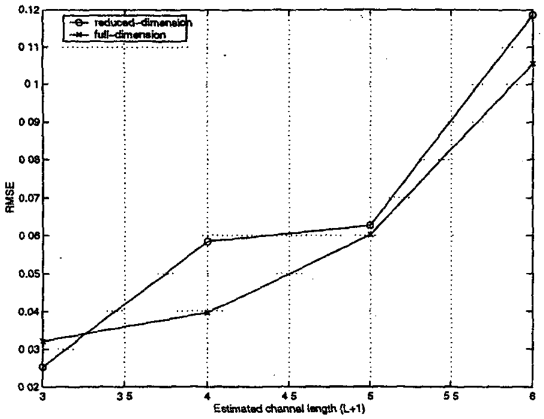

Figure 4. RMSE vs. estimated channel length

channel length $L + 1 = 3$ and all users ($K = 1, 2$ or 4) have same transmit power. The reduced complexity detec-

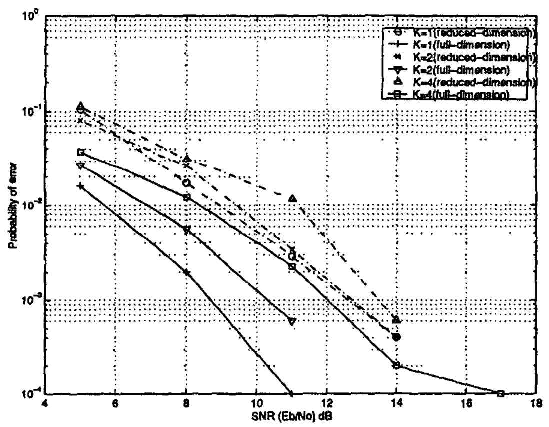

Figure 5. Bit Error Rate for the full and reduced dimension detector ($L + 1 = 3$)

tor results in higher BER than the full-dimension detector due to it's poorer channel estimation accuracy. However, at moderate SNR (14dB), the reduced complexity detector achieves (acceptable) BER below 10^{-3}. Note that the SNR penalty for the full vs. reduced-dimensioned detector decreases with increasing number of users - it is less than 4 dB for single user, about 2.2dB for $K = 2$ and less than 2dB when $K = 4$.

5 Conclusion

A new blind channel estimator for MC-CDMA systems that exploit the presence of virtual carriers was presented. The proposed detector architecture allows pre-filtering for dimensionality reduction in interests of reduced complexity at the expense of some performance degradation. Simulation results cataloged detector performance as a function of the relevant system parameters such as channel length and number of users. Future extension of this work will incorporate adaptivity into the detector to capture the time-varying (fading multipath) nature of typical wireless channel.

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