A SUBSPACE BLIND CHANNEL ESTIMATION METHOD FOR OFDM SYSTEMS WITHOUT CYCLIC PREFIX

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ABSTRACT

We propose a subspace based blind channel estimation method for OFDM systems over a time-dispersive channel. Our approach is motivated by the resemblance of the multichannel signal model resulting from oversampling (or use of multiple receive sensors) of the received OFDM signal to that in conventional single carrier system [1]. The proposed algorithm distinguishes itself from many previously reported channel estimation methods by the elimination of the cyclic prefix (CP), thereby leading to higher channel utilization. Comparison of the proposed method with other two reported subspace channel estimation methods [2, 3] is presented by computer simulations to support its effectiveness.

1. INTRODUCTION

Orthogonal Frequency-Division Multiplexing [4, 5] (OFDM) has been receiving considerable interest as a promising candidate for high-speed wireless/mobile communications systems due to its many advantages - notably, its high spectral efficiency, robustness to frequency selective fading, as well as the feasibility of low-cost transceiver implementations. It has already been used for digital audio/video broadcasting in Europe. As well, OFDM is being developed internationally for high-speed wireless LANs [6].

Channel estimation is indispensable to achieve coherent demodulation and consequently higher data rates. In practical OFDM systems operating over a time-dispersive channel, a Cyclic Prefix (CP) longer than the channel duration is usually inserted in the transmitted sequence to reduce the channel effect into a (complex) multiplicative distortion on each OFDM sub-channel in the frequency domain. In this means, appropriate training based approaches suffice to estimate the channel gains on each sub-channel as described in [7]; the estimate can be then used for gain/phase correction [4, 5]. The length of the CP is chosen for the maximum anticipated multipath spread; for IEEE 802.11a standard, this is 25% of an OFDM symbol duration, indicating a significant loss in utilization. Additionally, due to the time-varying nature of the channel, the training sequence needs to be transmitted periodically, causing further loss of channel throughput.

The disadvantage of the aforementioned training based channel estimation methods naturally stimulates the search for *blind* channel estimation methods that avoid the use of the training sequence or even the CP. Recently, the presence of the CP has been exploited for blind channel estimation [2, 3]. Specifically, Heath and Giannakis[2] proposed a spectrum fitting blind method based on the cyclostationarity property of the auto-correlation of the received data samples due to the CP insertion at the transmitter; this method however suffers from slow convergence of the estimator. Most recently, Cai and Akansu [3] developed a noise subspace method by utilizing the structure of the filtering matrix introduced by the CP insertion; it achieves faster convergence for smaller data records.

The main contribution of this paper is a blind subspace channel estimation algorithm which *avoids the use of the CP* (thus improving channel utilization) while achieving performance comparable to [3] with regards to estimator accuracy and convergence speed. However the method requires oversampling or receiver diversity, thereby increasing receiver cost /complexity. Nonetheless, typical oversampling factors of $M = 2$ are expected to be reasonable for implementation. Also, minimum mean-squared error (receiver) diversity combining has already been suggested to improve detection performance in [8] subsequent to channel estimation.

The paper is organized as follows: a baseband multichannel signal model for the OFDM system is introduced in Section 2. The subspace based channel estimator is developed in Section 3, along with the description of a sufficient condition on channel identifiability adjusted from [1]. Computer simulations are conducted in Section 4 to demonstrate the performance of the proposed algorithm with comparison to the two reported subspace methods [2] and [3]. Finally, Section 5 concludes the paper.

The notation used in this paper follows usual convention - vectors are denoted by symbols in boldface, $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$ are complex conjugate, transpose and conjugate transpose of $(\cdot)$, respectively. $ran(\cdot)$ gives the range of the matrix argument.

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2. SIGNAL FORMULATION

In this section, we describe a multichannel signal model for an OFDM system resulting from oversampling or multiple receiving sensors which closely resembles the model for single carrier system as in [1]. Consider an OFDM system as in Fig. 1 with Q sub carriers and *no* cyclic prefix extensions. The kth block of the 'frequency domain' information symbols is

$$\mathbf{s}(k) \quad = [s_0(k),\, s_1(k),\, \ldots,\, s_{Q-1}(k)]^T. \qquad (1)$$

For information symbol duration of T, the corresponding OFDM symbol duration $T_s = QT$. After multi-carrier modulation implemented by IFFT, the 'time domain' output signal vector is given by

$$\mathbf{x}(k) \quad = [x_0(k),\, x_1(k),\, \ldots, x_{Q-1}(k)]^T = \mathbf{W}_Q\, \mathbf{s}(k), \quad (2)$$

where $\mathbf{W}_Q$ is the $Q \times Q$-dimensional IDFT matrix

$$\mathbf{W}_Q \quad = \quad \frac{1}{\sqrt{Q}} \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & W_Q^{-1} & \cdots & W_Q^{-(Q-1)} \\ 1 & W_Q^{-2} & \cdots & W_Q^{-2(Q-1)} \\ \vdots & & & \vdots \\ 1 & W_Q^{-(Q-1)} & \cdots & W_Q^{-(Q-1)(Q-1)} \end{bmatrix}$$

with $W_Q = e^{-j2\pi/Q}$.

Each element of $\mathbf{x}(k)$ is then pulse shaped by $g_{tr}(t)$ to generate the continuous time signal sent on the channel

$$x(t) \quad = \quad \sum_{k=-\infty}^{+\infty} \sum_{p=0}^{Q-1} x_p(k) g_{tr}(t - pT - kT_s) \qquad (3)$$

2.1. Multichannel model for oversampling

Substituting $T_s = QT$ in (3) leads to

$$x(t) \quad = \quad \sum_{k=-\infty}^{+\infty} \sum_{p=0}^{Q-1} x_p(k) g_{tr}(t - (p + kQ)T) \qquad (4)$$

Thus denoting $q = p + kQ$, we identify $k = \lfloor \frac{q}{Q} \rfloor$ ($\lfloor x \rfloor$ is the largest integer contained in x) and $p = q$ *modulo* Q. Then the transmitted signal $x(t)$ can be rewritten as

$$x(t) \quad = \quad \sum_{q=-\infty}^{+\infty} x_q g_{tr}(t - qT) \qquad (5)$$

The signal $x(t)$ passes through a dispersive channel with impulse response $c(t)$ and is contaminated by AWGN noise $n(t)$, and is input into a front-end receive filter $g_{rx}(t)$.

Defining the composite channel filter $h(t) = g_{tr}(t) * c(t) * g_{rx}(t)$ and the filtered noise $v(t) = n(t) * g_{rx}(t)$ where $*$ denotes linear convolution, the received signal $r(t)$ is therefore

$$r(t) \quad = \quad \sum_{q=-\infty}^{+\infty} x_q h(t - qT) + v(t) \qquad (6)$$

Assume the composite channel $h(t)$ to have finite support $[0, (L+1)T)$ no longer than the OFDM symbol duration T_s; this implies that any ISI is only restricted to the past neighboring symbol as is generally true for OFDM. A synchronized rate M/T sampler (i.e. oversampling factor of M compared to information symbol sampling rate $1/T$) after $r(t)$ yields (for $m = 0, ..., M-1$)

$$\begin{aligned} r_i^{(m)} &= r(t_0 + iT + \frac{mT}{M}) \\ &= \sum_{l=0}^{L} x_{i-l} h(t_0 + lT + \frac{mT}{M}) + v_i^{(m)} \end{aligned} \qquad (7)$$

where $v_i^{(m)} = v(t_0 + iT + \frac{mT}{M})$.

Define $h^{(m)}(l) = h(t_0 + lT + \frac{mT}{M})$ and

$$\mathbf{h}^{(m)} = [h^{(m)}(L), \ldots, h^{(m)}(0)]^T, \qquad (8)$$

$$\mathbf{H}^{(m)} = Toeplitz([\mathbf{h}^{(m)T} \quad \underbrace{0 ... 0}_{(Q-L-1)\ zeros} \quad]). \qquad (9)$$

We process only the ISI-free samples[1] for channel estimation, i.e., the $M(Q-L)$ samples over the interval $(t_0 + (kQ+L)T)$ to $(t_0 + (k+1)QT - T/M)$; correspondingly, $r_i^{(m)}$'s for $i = kQ+L, \ldots, (k+1)Q-1$ where $m = 0, ..., M-1$ denotes the m-th sampling phase. Thus, the received signal for the m-th sampling phase corresponding to transmitted symbol $\mathbf{x}(k)$ is given by

$$\begin{aligned} \mathbf{r}^{(m)}(k) &= [r_{kQ+L}^{(m)}, \ldots, r_{kQ+Q-1}^{(m)}]^T \\ &= \mathbf{H}^{(m)} \underbrace{\begin{bmatrix} x_{kQ} \\ \vdots \\ x_{kQ+Q-1} \end{bmatrix}}_{\mathbf{x}(k)} + \underbrace{\begin{bmatrix} v_{kQ+L}^{(m)} \\ \vdots \\ v_{kQ+Q-1}^{(m)} \end{bmatrix}}_{\mathbf{n}^{(m)}(k)} \\ &= \mathbf{H^{(m)}} \mathbf{W}_Q \mathbf{s}(k) + \mathbf{n}^{(m)}(k). \end{aligned} \qquad (10)$$

Stacking all M $\mathbf{r}^{(m)}(k)$ ($m = 0, \ldots, M-1$) vectors yields

$$\begin{aligned} \mathbf{r}(k) &= [\mathbf{r}^{(0)T}(k), \ldots, \mathbf{r}^{(M-1)T}(k)]^T \\ &= \underbrace{\underbrace{\begin{bmatrix} \mathbf{H}^{(0)} \\ \vdots \\ \mathbf{H}^{(M-1)} \end{bmatrix}}_{\mathcal{H}} \mathbf{W}_Q}_{\mathcal{A}} \mathbf{s}(k) + \mathbf{n}(k) \\ &= \mathcal{A}\mathbf{s}(k) + \mathbf{n}(k) \end{aligned} \qquad (11)$$

The multichannel model (11) for OFDM *without* CP yields an equivalent filtering matrix $\mathcal{A}$ of dimension $M(Q-L) \times Q$.

[1]For $L << Q$ (which is plausible in several OFDM applications), the energy of the ISI samples is negligible comparing to that of the non-ISI affected samples.

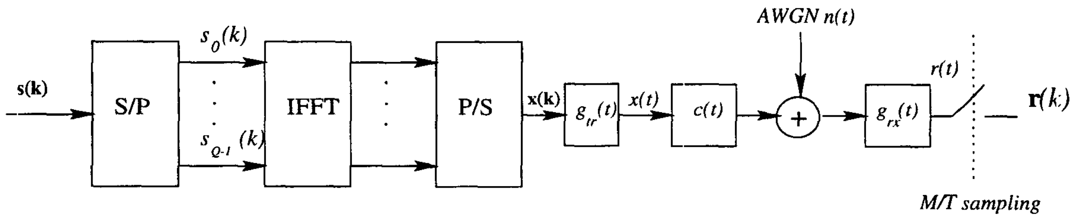

Fig. 1. Baseband OFDM system model

2.2. Multichannel model for multiple sensors

Instead of time oversampling, multiple receive sensors each sampled at rate $1/T$ also produce a multichannel signal model. However, in this situation, the signal $x(t)$ passes through *different* propagation channels and is received at an array of M sensors. Again assuming $[0, LT]$ support for all composite channels, we obtain $h^{(m)}(l) = h^{(m)}(t_0 + lT)$, and $\mathbf{h}^{(m)} = [h^{(m)}(L), \ldots, h^{(m)}(0)]^T$ as the discrete time equivalent channel impulse responses seen by the m-th sensor. As previously, collecting only the non ISI corrupted OFDM symbols at each sensor and stacking leads to a multichannel signal model similar to (11).

Remarks: The multichannel signal model is overdetermined when $(M-1)Q > ML$. When $Q >> L$ as assumed here, $M = 2$ typically suffices to satisfy the necessary condition for the subspace method and is assumed throughout the paper.

3. SUBSPACE BASED CHANNEL ESTIMATION

We now describe a subspace based channel estimator based on the structure of $\mathcal{A}$ shown in (11). The time index k is omitted when there's no confusion.

3.1. Sufficient Conditions for Identifiability

From (11), $\mathcal{A} = \mathcal{H}\mathbf{W}_Q$ $(M(Q-L) \times Q)$ where $\mathcal{H}$ has the same *Toeplitz* structure as the filtering matrix $\mathcal{H}_N$ in [1] and $\mathbf{W}_Q$ is unitary. A sufficient condition for channel identifiability follows as a corollary of *Theorem 1 and 2* in [1].

Theorem 1: [1] $\mathcal{H}$ is full column rank, i.e., $rank(\mathcal{H}) = Q$, if i) the polynomials $H^{(m)}(z) \stackrel{def}{=} \sum_{j=0}^{L} h_j^{(m)} z^j$ have no common zero; ii) $Q \geq L$; and iii) at least one polynomial $H^{(m)}(z)$ has degree L.

Since $\mathbf{W}_Q$ is unitary, this directly leads to $rank(\mathcal{A}) = rank(\mathcal{H}\mathbf{W}_Q) = rank(\mathcal{H})$. Therefore, the above conditions for $\mathcal{H}$ to be full column rank also guarantee that $\mathcal{A}$ is full column rank.

Assume the user's transmitted information symbols $s_i(k)$'s to be i.i.d. sequences with zero mean and known variance σ_s^2 (σ_s^2 can be set to unity, without loss of generality). Also assume the Nyquist pulse shaping is employed so that each element of $\mathbf{n}(k)$ in (11) is AWGN with zero mean and variance σ_n^2. After collecting N_b signal vectors, we have

$$\mathbf{Y} = \mathcal{A}\underbrace{[\mathbf{s}(1), \ldots, \mathbf{s}(N_b)]}_{\mathbf{S}} + \mathbf{N} = \mathbf{X} + \mathbf{N} \qquad (12)$$

The Singular Value Decomposition (SVD) on the unpurturbed received signal matrix $\mathbf{X}$ yields

$$\mathbf{X} = \begin{bmatrix} \mathbf{U}_s & \mathbf{U}_n \end{bmatrix} \begin{bmatrix} \mathbf{\Sigma}_s & \\ & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{V}_s^H \\ \mathbf{V}_n^H \end{bmatrix} \qquad (13)$$

where $[\mathbf{U}_s, \mathbf{U}_n]$ is an $M(Q-L) \times M(Q-L)$ unitary matrix. The Q columns of $\mathbf{U}_s$ span the signal subspace, while $M(Q-L) - Q$ column vectors of $\mathbf{U}_n$ span a subspace (often known as the noise subspace as in practice the SVD is applied on the noise purturbed signal matrix $\mathbf{Y}$) orthogonal to the signal subspace. $\mathbf{\Sigma}_s = diag(\lambda_1, \lambda_2, \ldots, \lambda_Q)$ is a diagonal matrix consisting of Q significant singular values corresponding to the signal subspace. The orthogonality property between signal subspace and noise subspace asserts

$$\mathbf{U}_n(i)^H \mathcal{A} = 0 \ (i = 1, \ldots, M(Q-L) - Q) \qquad (14)$$

where $\mathbf{U}_n(i)$ is the i-th column of $\mathbf{U}_n$.

The uniqueness of the estimate of $\mathbf{h}^{(0)}, \ldots, \mathbf{h}^{(M-1)}$ based on (14) can be obtained as corollary of Theorem 2 in [1].

Theorem 2: [1] Let $\mathbf{h} = [\mathbf{h}^{(0)T}, \ldots, \mathbf{h}^{(M-1)T}]^T$, and $\mathbf{h}'$ be a $M(L+1) \times 1$ vector distinct from $\mathbf{h}$; filtering matrix $\mathcal{H}$ and $\mathcal{H}'$ are constructed using $\mathbf{h}$ and $\mathbf{h}'$ respectively. When $Q \geq L$, if $ran(\mathcal{H}') = ran(\mathcal{H})$, then $\mathbf{h}' = \alpha\mathbf{h}$ where α is a scalar.

It is easy to show that $ran(\mathcal{A}) = ran(\mathcal{A}')$ also gives the uniqueness of the channel estimation $\mathbf{h}$ from (14) for the OFDM case, where now $\mathcal{A} = \mathcal{H}\mathbf{W}_Q$ and $\mathcal{A}' = \mathcal{H}'\mathbf{W}_Q$ are constructed using $\mathbf{h}$ and $\mathbf{h}'$ respectively. Since unitary matrix does not change the range of $\mathcal{H}$, therefore, if $ran(\mathcal{A}) = ran(\mathcal{A}')$, then $ran(\mathcal{H}') = ran(\mathcal{H})$, and consequently $\mathbf{h}' = \alpha\mathbf{h}$.

In summary, the *sufficient condition* for channel identifiability in the OFDM system of interest is:
i) the polynomials $H^{(m)}(z) \stackrel{def}{=} \sum_{j=0}^{L} h_j^{(m)} z^j$ have no common zero; ii) $Q \geq L$; iii) at least one polynomial $H^{(m)}(z)$ has degree L.

Remarks:
1) The application of noise subspace method in the OFDM of

interest is a special case of [1]. Note that the requirement $Q \geq L$ is generally satisfied in practice for typical OFDM system and channel delay spreads.

2) Comparing to the other noise subspace method [3] which is not sensitive to channel order overestimation, the proposed method requires a good estimation on the channel order L.

3.2. Blind Channel Estimator

Let

$$\mathbf{U}_n(i) = [u_i(0), \ldots, u_i(MQ - ML - 1)]^T \quad (15)$$

Exploiting the *Toeplitz* structure of $\mathcal{H}$ yields

$$\mathbf{U}_n(i)^T \mathcal{H} = \mathbf{h}^T \mathcal{U}_i, \quad (16)$$

where the $M(L+1) \times Q$ matrix $\mathcal{U}_i = [\mathcal{U}_i^{(0)T}, \ldots, \mathcal{U}_i^{(M-1)T}]^T$ is generated from vector $\mathbf{U}_n(i)$, and each $(L+1) \times Q$ submatrix $\mathcal{U}_i^{(m)}$ $(m = 0, \ldots, M-1)$ is formed as

$$\mathcal{U}_i^{(m)} = Toeplitz([u_i(m(Q-L))...u_i((m+1)(Q-L)-1)])$$

Thus, by defining $\tilde{\mathbf{h}} = (\mathbf{h})^*$ and $\mathcal{G} = [\mathcal{U}_1, \ldots, \mathcal{U}_{M(Q-L)-Q}]^*$, (14) suggests the channel estimator

$$\hat{\tilde{\mathbf{h}}} = \arg \min_{||\tilde{\mathbf{h}}||=1} \tilde{\mathbf{h}}^H \hat{\mathcal{G}} \hat{\mathcal{G}}^H \tilde{\mathbf{h}} \quad (17)$$

where $\hat{\mathcal{G}}$ is the estimate of $\mathcal{G}$. It is well known that $\tilde{\mathbf{h}}$ (or equivalently $\mathbf{h}$) is the eigenvector corresponding to the smallest eigen value of the matrix $\hat{\mathcal{G}}\hat{\mathcal{G}}^H$.

4. SIMULATION RESULTS

Monte Carlo simulations were conducted to assess the effectiveness of the proposed blind estimator with comparison to other two reported subspace channel estimation methods for OFDM [2, 3]. The proposed method eliminates the CP, but requires multiple sensors or oversampling at the receiver; we assume $M = 2$. The methods of [2, 3] used as comparison baselines require CP; thus results from them are obtained based on one sensor and usual sampling rate (rate 1/T; i.e. no oversampling) with the length of the CP set to a quarter of the available sub carriers Q, as in [2, 3].

To evaluate the estimation error, the normalized Root Mean Squared Error(RMSE)

$$\text{RMSE} = \frac{1}{||\mathbf{h}||} \sqrt{\frac{1}{N_m M(L+1)} \sum_{p=1}^{N_m} ||\hat{\mathbf{h}}_p - \mathbf{h}||^2}$$

is used, where the subscript p refers to the p-th simulation run and N_m denotes the number of runs. Information sequence $s_i(k)$'s are BPSK modulated. Input SNR is defined as SNR $\stackrel{def}{=}$ $10 \log_{10} \frac{\sigma_s^2}{\sigma_n^2}$. Time-invariant multipath channel with order $L = 3$ is generated according to Hoeher's method [9] by setting maximum Doppler shift to 0. The channel coefficients are listed below:

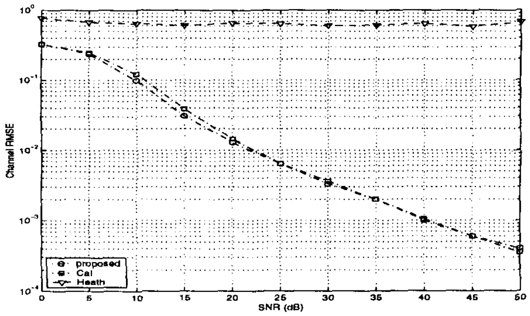

Fig. 2. Channel error versus SNR

$$\mathbf{h}^{(1)} = [(-0.1892, 0.4273), (-0.2839, 0.6984), (0.1274, 0.4321), (-0.0451, 0.0912)]^T,$$
$$\mathbf{h}^{(2)} = [(0.3600, 0.1388), (0.1041, 0.4126), (0.0914, 0.1885), (0.2052, -0.0739)]^T.$$

Results shown are the average over $\mathbf{h}^{(1)}$ and $\mathbf{h}^{(2)}$ for all three methods. And 100 runs are carried out to obtain the average, i.e., $N_m = 100$.

Example 1: In Fig. 2, we examine the estimator error as a function of the input SNR's and compare it with the results from other two CP-based methods, using the following setup[2]: $Q = 15$ and $N_b = 120$. It can be seen that both our approach and the method in [3] (marked as Cai) perform much better than that of [2] (marked as Heath), reflecting the fast convergence property of the noise subspace estimator for small data record. Also note that the estimator error of our proposed method is close to the Cai method for comparable computation complexity.

Example 2: In the second example, with the same Q and the same channel setup as before, we illustrate the estimator error as a function of the number of data blocks N_b. For SNR $= 15$dB, Fig. 3 shows that the estimation accuracy improves as the number of data blocks increases for all three subspace methods. Note that the noise subspace methods (both ours and Cai's) achieve low estimate error (< 0.1) with only 60 OFDM blocks, while the spectrum fitting method (Heath's) requires more than 1000 OFDM blocks for comparable performance. The superior performance of the noise subspace method over the spectrum fitting method makes them a possible candidate for wideband communication scenarios where the channel is time-invariant for only a few OFDM symbols. Moreover, the proposed method avoids the CP and therefore leads to higher throughput than [3].

[2]For the CP-based methods, the length of the CP is set to 4 in this example.

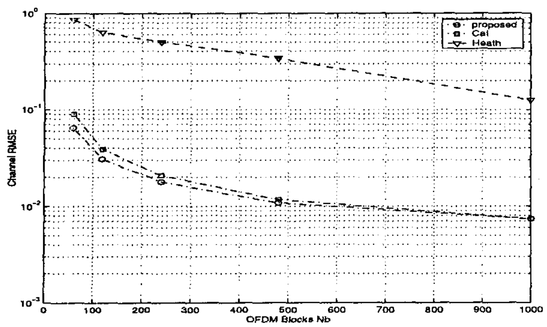

Fig. 3. Channel error versus number of data blocks N_b (simulation results, SNR $= 15dB$)

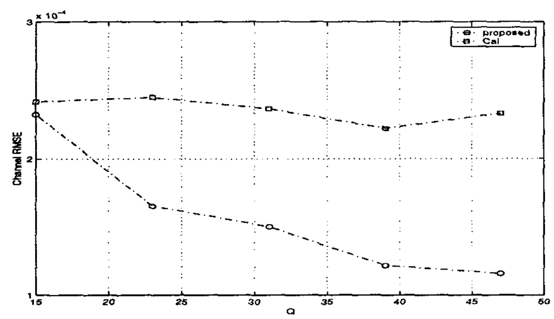

Fig. 4. RMSE vs. Q

Example 3: Finally, the effect of varying Q (meaning longer OFDM symbol duration) on the estimator error for the two noise-subspace methods (proposed and Cai) is investigated in Fig. 4 with SNR $= 40dB$, $N_b = 2000$, and Q varying from 15 to 47 (the length of the CP is set to $(Q+1)/4$ and changed accordingly for Cai's method). Larger Q means larger dimension of the noise subspace ($(M-1)Q - L$ for the proposed method and $\frac{MQ}{4} - L$ for Cai's), yielding more constraints on the channel vector (as in (14)), and thus leads to improvement in the channel estimate. Also note that for the proposed method, the noise subspace dimension increases faster with Q than it does for Cai's method, leading to larger performance improvement.

5. CONCLUSION

In this paper, we presented a subspace based blind channel estimator for OFDM system without the CP. A sufficient condition on identifiability was also developed. The algorithm is attractive for its potential to increase the system's channel utilization due to the elimination of the CP. Comparison of the proposed method with other two reported subspace channel estimation methods by computer simulations illustrates the superior performance of the proposed method with regard to both the estimate accuracy and the speed of convergence.

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