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Fixed channel assignment algorithm for multi-radio multi-channel MESH networks

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Summary

Recently, multi-radio mesh technology in wireless networks has been under extensive research. This is because of its potential of overcoming the inherent wireless multi-hop throughput, scalability and latency problems caused by the half-duplex nature of the IEEE 802.11. The concept of deploying multiple radios in wireless network access points (APs) has shown a promising way to enhance the channel selection and the route formation while the MESH topology allows more fine-grained interference management and topology control. Within this realm, given a set of end-to-end objectives, there are multiple issues that need to be identified when we consider the optimization problem for fixed multi-channel multi-hop wireless networks with multiple radios. This paper addresses the static channel assignment problem for multichannel multi-radio static wireless mesh networks. We first discuss its similarities and differences with the channel assignment problem in cellular networks (WMN). Next, we present four metrics based on which mesh channel assignments can be obtained. Three of these metrics attempt to maximize simultaneous transmissions in a mesh network, either directly or indirectly. The fourth metric quantifies the 'diversity' of a particular assignment and can be used as a secondary criterion to the other three metrics. Related optimization models have also been developed. Copyright © 2007 John Wiley & Sons, Ltd.

KEY WORDS: MESH networks; optimization; wireless networks; ILP

1. Introduction

The emergence of cost-effective wireless access client devices (such as 802.11) has made broadband access for (mobile) end-users an emerging reality. The success of 802.11 and expected availability of 802.16 imply that wireless access is set to increase in both geographic scale (wide-area) and also enhanced user density. Clearly, the increased demand for aggregate network throughput imply that spectrum scarcity[‡] will have to be mitigated by other means, namely via a combination of architectural and design innovations, particularly at the lower layers of the protocol stack.

Within this context, two trends are particularly noteworthy; one is the proposal for a Layer-2 mesh or multi-hop network currently under definition within the IEEE 802.11s Task Group. The architecture assumes a set of wirelessly inter-connected nodes (which comprise both traditional access points (APs) that allow

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‡ Currently, only a very limited number of such orthogonal channels are available: 3 in 802.11b (2.4 GHz band) and 8–12 in 802.11a (5 GHz band).

direct client access and future 'routers' that only relay packets between other mesh elements and do not have clients) to form a multi-hop network. The second is the availability of client and infrastructure devices equipped with multiple radios. For example, cellular phones may have a cellular 2.5/3G and bluetooth radio interfaces (and soon 802.11 radios as well), exemplifying an element in a *heterogeneous* (or multi-protocol) network. Soon, laptops and access points with multiple 802.11 WLAN adapters are anticipated (e.g., with PCI slots for two 802.11a radios in addition to a 802.11b/g radio) that can form a *homogeneous* (or single protocol) network. The conjunction of these two factors have significant impact as explained next in solving a fundamental problem: scaling aggregate end-to-end throughput with network size.

There are a number of common issues involved in traditional multi-hop wireless networks. These, as was noted in References [1,2], include efficient methods for sharing the common radio channel, network connectivity, network capacity, and methods for managing and controlling the distributed network. A particular issue that is of interest to us is the channel assignment problem in multi-hop wireless networks with a single radio. In particular, it is well-known that in single radio multi-hop wireless networks, the end-to-end throughput on a route drops as the number of hops increase. A primary reason is due to the fact that a single wireless transceiver operates in half-duplex mode, that is, it cannot transmit and receive simultaneously. An incoming frame must therefore be received fully before the node can switch from receive mode to transmit mode. Consequently, for a linear chain topology of n nodes where only one transmission is allowed at a time in the network,[§] the aggregate throughput $\sim O(\frac{1}{n})$. Note that this is irrespective of the number of available orthogonal channels; the aggregate throughput can only improve if some form of spatial reuse of the channels is achieved, whereby multiple simultaneous transmissions can occur.

The idea of increasing network capacity and utilization is by exploiting the use of multiple channel and channel reuse opportunities has been extensively researched in literature. Early work by Cidon and Sidi [3] presented a distributed dynamic channel assignment algorithm that is suitable for shared channel multi-hop networks. Hajek and Sasaki [4] studied the network as an arbitrary undirected graph and presented two polynomial time algorithms for link scheduling. They also addressed the problem of joint routing and scheduling to satisfy end-to-end demand. They showed that, under a certain simplifying assumption, the routing and scheduling problems can be decoupled to a large extent, without increasing the schedule length. Their algorithm complexity was later improved by Ogier in Reference [5] with a more efficient approach. When using an arbitrary graph modeling approach, many scheduling problems were shown by researchers to be NP-hard [6–8].

Recently, Gupta and Kumar [9] investigated the asymptotic capacity problem of a multi-hop *fixed* wireless network and showed that in arbitrary network the network capacity for n identical nodes, each node is capable of transmitting W bps, scales as $\Theta(W\sqrt{n})$ bmps, and for a random network, the network capacity scales as $\Theta(W\sqrt{\frac{n}{\log n}})$ bps. The network model in Reference [9] was later extended by Grossglauser and Tse in Reference [10] to include mobility and independent movement of users around the network and showed that delay tolerant application can take advantage of network mobility to significantly increase network throughput and capacity. In particular, under the assumptions of loose delay and one-hop relaying node, they showed that the throughput of the mobile ad hoc network is $O(n)$. While References [9,10] assume *single* channel, single radio per node networks, a related work by Kyasanur and Vaidya in Reference [11] studies the asymptotic capacity for the multichannel multi-radio fixed wireless network. They show that, given the number of channels c and the number of interfaces per node m, the capacity is dependent on the ratio $\frac{c}{m}$, and not the exact values of either c or m.

As an illustrative example, consider the transmission $1 \rightarrow 2 \rightarrow 3$ in Figure 1(a). With one radio, node 2 spends roughly half the time receiving from node 1 and half the time transmitting to node 3. Consequently, if the source (node 1) rate is R bps, the average receive rate at node 3 is approximately $R/2$ bps. With two radios at node 2 and two orthogonal channels, radio 1 can be tuned to channel 1 and radio 2 can be tuned to channel 2, in which case the receive rate at node 3 will be theoretically equal to R bps. Now, consider the case when there is a concurrent[‖] transmission on the route $4 \rightarrow 2 \rightarrow 5$. In this case, node 2 has to spend a quarter of its time receiving from nodes 1 and 4 and

[§] This is justified when the carrier sensing (CS) range is sufficiently large or the network size is sufficiently small.

[‖] By concurrent transmission, we refer to the case when the sending nodes are trying to access the channel at the same time.

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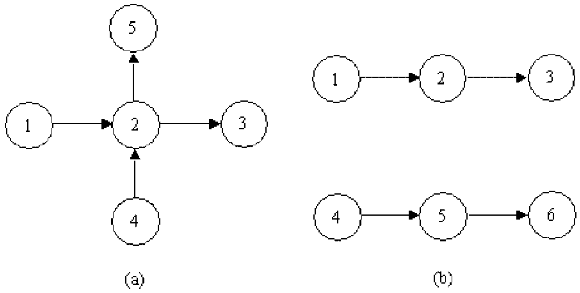

Fig. 1. An example motivating the improvement in throughput that can be obtained with multiple radios and/or multiple channels: (a) With one radio at node 2, each of the two flows, $1 \rightarrow 2 \rightarrow 3$ and $4 \rightarrow 2 \rightarrow 5$, receive an end-to-end throughput of $R/2$ bps (where R is the source rate) if they are scheduled at different times. However, if the two flows are simultaneous, the receive rate for both flows drops to $R/4$ bps. With two radios and availability of two orthogonal channels, the receive rate for both flows increases to $R/2$ bps, the same as each flow would have received if they were scheduled at different times; (b) An illustration of a scenario when having multiple orthogonal channels is helpful even with one radio. For example, if two channels are available, one each can be used for the two transmissions. The receive throughput for each flow in this case is $R/2$ bps.

transmitting to nodes 3 and 5. The average receive rate at nodes 3 and 5 in this case is $R/4$ bps. Having multiple nonoverlapping channels does not help in this specific scenario since the limiting factor is the availability of only one radio at node 2 (an example of where multiple channels would help even with a single radio is shown in Figure 1(b). Next, let us consider the case when node 2 is equipped with two radios and there are two available orthogonal channels. In this case, radio 1 can be tuned to channel 1 and radio 2 can be tuned to channel 2. If radio 1(2) is used on a half-duplex mode to support the route $1 \rightarrow 2 \rightarrow 3$ ($4 \rightarrow 2 \rightarrow 5$), the average receiver throughput for each flow doubles to $R/2$ bps, the same as each flow would have received if they were scheduled at different times.

The preceding example illustrates the throughput benefits that can be obtained with *multiple radios and multiple channels*. There are several interesting research issues in the context of multi-radio, multichannel wireless mesh networks (WMN); finding the *optimum channel assignment* for a given number of radios per node and a given number of orthogonal channels is the objective in this work. In particular, we propose in this paper four metrics which can be used for finding the optimal channel assignment and discuss integer linear programming (ILP) models which can be used for solving the related optimization problems. Following the approach that was presented in Reference [12], we formulate our optimization models to include different optimality objectives. These objectives are indirect metrics which are intended to improve the utilization of a network by shrinking the interference domains of the nodes, thereby allowing more simultaneous transmissions to occur. Specifically, we first consider minimization of the average size of a co-channel interference set. Then, we extend it to the case when the maximum (or bottleneck) size of a co-channel interference set is to be minimized. In addition, we also include the *channel diversity* as an important criterion for channel assignment optimization problem.

The remainder of the paper is organized as follows. In Section 2, we discuss related prior work and in Section 3, we outline our network model and assumptions. In particular, we discuss node-based versus edge-based channel assignment approaches and propose four metrics for static channel assignment in multi-radio, multichannel mesh networks. Optimization models for solving the related channel assignment problems are developed in Sections 4 and 5. Sections 6 and 7 present our simulation results and conclusion respectively.

2. Related Work

Several other studies on the subject of *multi channel* multi-hop wireless networks can be found in References [13–16], for example, MAC protocols based on modification of IEEE 802.11 were proposed for utilizing multiple channels. while the aforementioned studies requires modification to the IEEE 802.11, Bahl and Dunagan [17] propose a new protocol, slotted seeded channel hopping (SSCH), that extends the benefits of channelization to ad-hoc networks. Their protocol runs over unmodified IEEE 802.11 with a single interface. The SSCH protocol operates at the link layer, but it can be implemented in software over an IEEE 802.11-compliant wireless Network Interface. They show through extensive simulations that their proposed scheme yields significant capacity improvement in a variety of single hop and multi-hop wireless scenarios.

Multi-radio multichannel technology provide great potential as well as new challenges in network design and implementation. Accordingly, in recent years, a great amount of research has been devoted to this subject. In References [18–22], to cite a few, the routing and channel assignment problems in multi-hop multi-radio mesh networks have been considered. In particular, Raniwala *et al.* [21,22] propose a

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centralized load-aware joint channel assignment and routing algorithm, which is constructed with a multiple spanning tree-based load balancing routing algorithm that can adapted to traffic load dynamically. They demonstrate the dependency of the channel assignment on the load of each virtual link, which in turns depends on routing. They also show that the problem of channel assignment is NP-hard. In Reference [20], the multi-radio mesh network under the assumption that the network has the ability to switch an interface from one channel to another dynamically was investigated. A distributed interface assignment strategy that accounts for the cost of interface switching was presented. Their routing strategy selects routes which have low switching and diversity cost taking into account the global resource usage to maximize the network utilization and allows the nodes to communicate without any specialized coordination algorithm.

A different aspect of the multi-hop multi-radio mesh networks was investigated by Draves *et al.* [19]. They propose a new routing metric weighted cumulative expected transmission times (WCETT) which is implemented in a routing protocol called multi-radio link-quality source routing (MR-LQSR), in which all nodes are assumed to be stationary and the channel assignment is predetermined. The WCETT metric is aimed at achieving a tradeoff between delay and throughput by balancing the usual criterion (minimum number of hops) with the notion of *channel diversity*. Their results show that classical shortest path routing is not suitable when multiple radios are deployed; exploiting channel diversity in multi-radio mesh networks significantly improves the network capacity and makes better utilization of the channel resources.

3. Network Model and Assumptions

In this section, we outline our network model and assumptions.

(1) We consider a *static* N-node wireless mesh network. Several applications have been proposed in the literature for WMN; see for example [23]. Of these, infrastructure meshes for *community and neighborhood networking* is one example where the network can be assumed to be static or minimally mobile.
(2) Node n has K_n radios, $K_n \geq 2$. We do not require all nodes to have the same number of radios.
(3) Given a target probability of bit error rate from node i to node j, the transmitter power at node i necessary to support the link $i \rightarrow j$,[¶] $\mathbf{P}_{ij}$, is proportional to $P_0 d_{ij}^{\alpha}$, where d_{ij} is the Euclidean distance between nodes i and j, α is the channel loss exponent, typically between 2 and 4, and P_0 is the reference power when $d_{ij} = 1$ m. Without any loss of generality, we set the proportionality constant equal to 1 and therefore:

$$\mathbf{P}_{ij} = P_0 d_{ij}^{\alpha} \tag{1}$$

(4) We assume that $\mathbf{P}_{ij} = \mathbf{P}_{ji}, \forall (i, j) \in \mathcal{N}$ where $\mathcal{N}$ is the set of all nodes in the network and $N = |\mathcal{N}|$.
(5) There is a constraint on the maximum power level per sector which a node can use for transmission and that this parameter is identical for all nodes. We denote this maximum power level by $P^{\max}$.
(6) Let $\mathcal{E}$ be the set of all bidirected edges in the network and $E = |\mathcal{E}|$. Using the transmitter power constraint, the set $\mathcal{E}$ is given by:

$$\mathcal{E} = \{(i \leftrightarrow j) : (i \neq j) \in \mathcal{N}, P^{\max} \geq \mathbf{P}_{ij}\} \tag{2}$$

The third condition on the right hand side of Equation (2) enforces bidirectionality of edges based on the maximum power constraint. Note that because of our assumption that the matrix $\mathbf{P}$ is symmetric, $P^{\max} \geq \mathbf{P}_{ij} \Rightarrow P^{\max} \geq \mathbf{P}_{ji}$. For the sake of notational simplicity, we will also use the set $\mathcal{E}$ to refer to all directed edges, $\{i \rightarrow j\}$, in the graph, since:

$$(i \leftrightarrow j) \in \mathcal{E} \Leftrightarrow (i \rightarrow j) \in \mathcal{E} \;\&\; (j \rightarrow i) \in \mathcal{E} \tag{3}$$

We will refer to the graph $G = (\mathcal{N}, \mathcal{E})$ as the *reachability graph* of the network.
(7) We assume that the nodes do not apply any transmitter power control. Given a desired reachability graph, the transmit power level of any node i is therefore assumed to be adequate to reach all its neighbors in the presence of noise only.

Given the typically high traffic loads in cellular networks and limited availability of the radio spectrum, it is impractical to assign different frequencies to

[¶] We use the notation $\{i, j\}$ to represent the node pair. A directed link from i to j will be represented by $i \rightarrow j$ and an undirected (or, bidirected) link between i and j will be represented by $i \leftrightarrow j$.

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all ongoing communications at any time.[#] Channels are therefore reused across the network. However, channel reuse introduces interference which impairs the signal quality of ongoing transmissions. Careful consideration must therefore be given as to how the channels are reused in the network. Broadly speaking, channel assignment constraints can be classified as:

(1) *Co-channel reuse constraints*: These constraints stipulate the minimum physical separation[**] required between a pair of base stations such that the mutual interference is within acceptable limits when a common channel is being used simultaneously for transmission in the corresponding cells.
(2) *Adjacent channel use constraints*: These constraints stipulate that adjacent frequency bands (even if they are non-overlapping) in the radio spectrum may not be used simultaneously by nearby cells or within the same cell[††] Adjacent channel constraints are motivated by filter imperfections which allow adjacent frequencies to 'bleedover' into the passband of the unintended receivers. Further details on co-channel and adjacent channel interference can be found in Reference [24].

Using the above constraints, a number of optimization problems can be defined for obtaining a suitable channel allocation. For example, the objective function could be to maximize the spatial reuse of frequencies, which is closely related to the graph coloring problem.[‡‡] This problem as also known as the *minimum order frequency assignment problem* (FAP). Other possible objective functions are:

- Maximization of the number of frequencies that can be assigned to any cell. This is also known as the *maximum service FAP*.
- Minimization of the difference between the maximum and minimum frequencies used in the assignment scheme. This is also known as the *minimum span FAP*.
- Minimization of the average call blocking probability.
- Maximization of the total traffic carried by the cells [26].

A survey of the optimization formulations for the first three objectives discussed above can be found in Reference [27] and the references cited therein.

Channel assignment in WMN can be classified as being (i) static, (ii) dynamic, or (iii) hybrid.[§§] A static assignment scheme assigns channels to the radio interfaces, either permanently, or for *long durations* relative to the channel switching time [20]. This scheme is the easiest to implement and no additional switching delay is incurred during data communication. In a dynamic assignment scheme, on the other hand, radios are allowed to switch channels frequently. While this may allow efficient spectrum utilization with fewer radio interfaces, sophisticated and precise coordination algorithms are required to be implemented so that two nodes who wish to communicate with each other can tune one of their interfaces to a common channel. In an heterogeneous network where some radios may have higher switching delays than the others, the coordination algorithm must also take into account relative differences in switching delays between a (transmitter, receiver) pair. Moreover, there is the possibility of high switching delays along some end-to-end paths which may be an important factor in delay critical applications. Finally, in a hybrid channel assignment scheme, a subset of the available channels and radios are assigned statically and the rest of the spectrum is available for dynamic assignment to the other radios.

In this paper, we consider the static (or the static portion of a hybrid scheme) channel assignment problem on a network of N nodes. As with cellular networks, we believe that the simplicity of the static scheme is a primary factor which tips the scales in its favor as far as practical implementation is concerned. The network is allowed to be heterogeneous in the sense

[#] We are implicitly assuming FDMA multiple access. However, the same ideas are equally valid for other multiple access schemes such as TDMA or hybrid TDMA/FDMA (as in GSM).

[**] The physical separation is influenced by several parameters such as the radius of each cell, distance between the centers of the two cells, the transmitter power and the actual frequency. For example, signals using a carrier frequency of 1800 MHz fade faster than 900 MHz signals and therefore a smaller co-channel reuse distance can be used for the former.

[††] Some authors prefer to use the term *co-site separation constraint* to refer to adjacent channel interference within the same cell.

[‡‡] An excellent discussion of the relationship between frequency assignment and graph coloring problems can be found in Reference [25].

[§§] A similar classification exists for channel assignment in cellular networks. See Reference [28] for a survey.

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that all nodes are not required to have the same number of radio interfaces.

Given an assignment of channels to radios (*radio based channel assignment*), nodes i and j can communicate if they share a common channel and are within transmission range of each other. If these conditions are satisfied, we say that a *link* (*edge*) exists between i and j. One of the goals of the channel assignment problem is to ensure proper radio connectivity. In other words, if an edge exists between nodes i and j in the reachability graph, there must be one radio interface on i and j which are assigned the same channel, say f. Channel f is then used by nodes i and j for both transmission and reception. As an alternate to radio based channel assignment, we can adopt an *edge based channel assignment* scheme in which a channel is assigned to all edges of the reachability graph. In this case, we do not need to worry about connectivity between nodes i and j since the channel assigned to the edge between i and j, say f, will be used by the nodes for communicating with each other. However, with an edge based assignment method, we need to make sure that the total number of unique channels assigned to a set of links which are incident on any node n does not exceed the number of radio interfaces at node n, K_n.[|||] This constraint is necessary since channel switching is not allowed in a static scheme. In this paper, we adopt the latter approach and assign channels to edges. A channel assignment which allocates channels to all edges in the reachability graph, while satisfying the *radio limitation constraint* discussed above, is defined to be *feasible*. Figure 2 illustrates the radio versus edge based channel assignment schemes.

The preceding paragraph demonstrates why a random assignment of channels, either to radios or to edges, may not yield a feasible solution. With the radio based scheme, if radio $A1$ chooses frequency F_1 and radio $C1$ chooses frequency F_2 in Figure 2, the two radios of B must be tuned to F_1 and F_2 (F_3 cannot be a choice) to ensure that it can communicate with nodes A and C. Additionally, radio $D1$'s choice is also limited between F_1 and F_2 as otherwise a communication link would not exist between nodes B and D. With the edge based assignment scheme, if the edge $A \leftrightarrow B$ chooses F_1 and $B \leftrightarrow C$ chooses F_2, $B \leftrightarrow D$ must choose between F_1 and F_2 so as not to violate the 2-radio limitation on node B.

[|||] Note that this constraint is not necessary with the radio based assignment scheme.

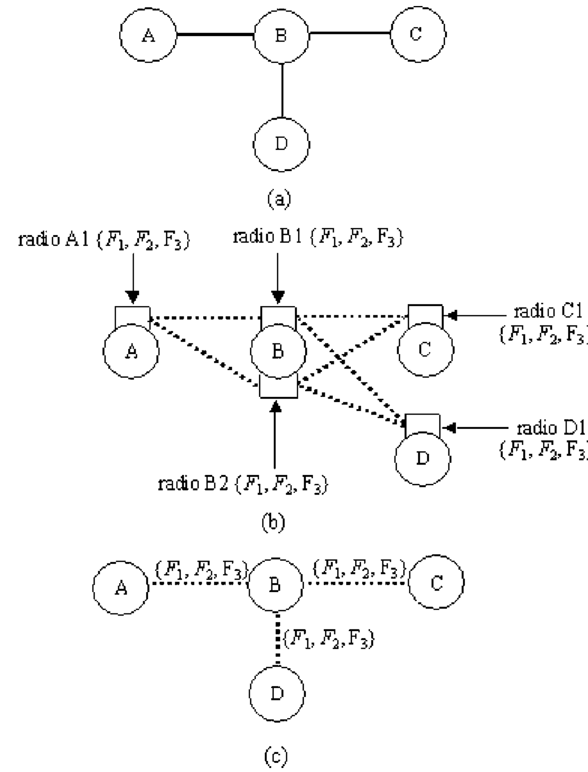

Fig. 2. Illustrating radio based versus edge based channel assignment schemes: (a) Desired reachability graph. Nodes A, C and D have a single radio each while node B has 2 radios. The radios are numbered $A1$, $B1$, $B2$, $C1$ and $D1$. Assume that 3 orthogonal channels are available for data communication; (b) In a radio based assignment scheme, channels are assigned to individual radios. In order to ensure radio connectivity between nodes A and B, they must share a common channel. Similarly, (B and C) and (B and D) must share a common channel to ensure radio connectivity; (c) In an edge based assignment scheme, channels are assigned to the edges of the reachability graph. In this example, since node B has only two radios, only two of the three outgoing edges from B can have distinct channels.

We now look at the interference pattern in a 802.11 wireless network under the assumption that all nodes in the mesh employ the RTS/CTS mechanism to combat the hidden terminal problem before actual data transmission. When a single channel is available (which is what the IEEE 802.11 MAC protocol is designed for), after a successful RTS/CTS exchange between a pair of nodes, no node within CS range of the transmitter and receiver can communicate for the duration of the subsequent data packet. Figure 3(b) shows the set of 22 links (which is almost 50% of the total number of edges) in a 5×6 grid that must remain silent when the edge $a \leftrightarrow b$ is active (i.e., either a is transmitting to b or receiving

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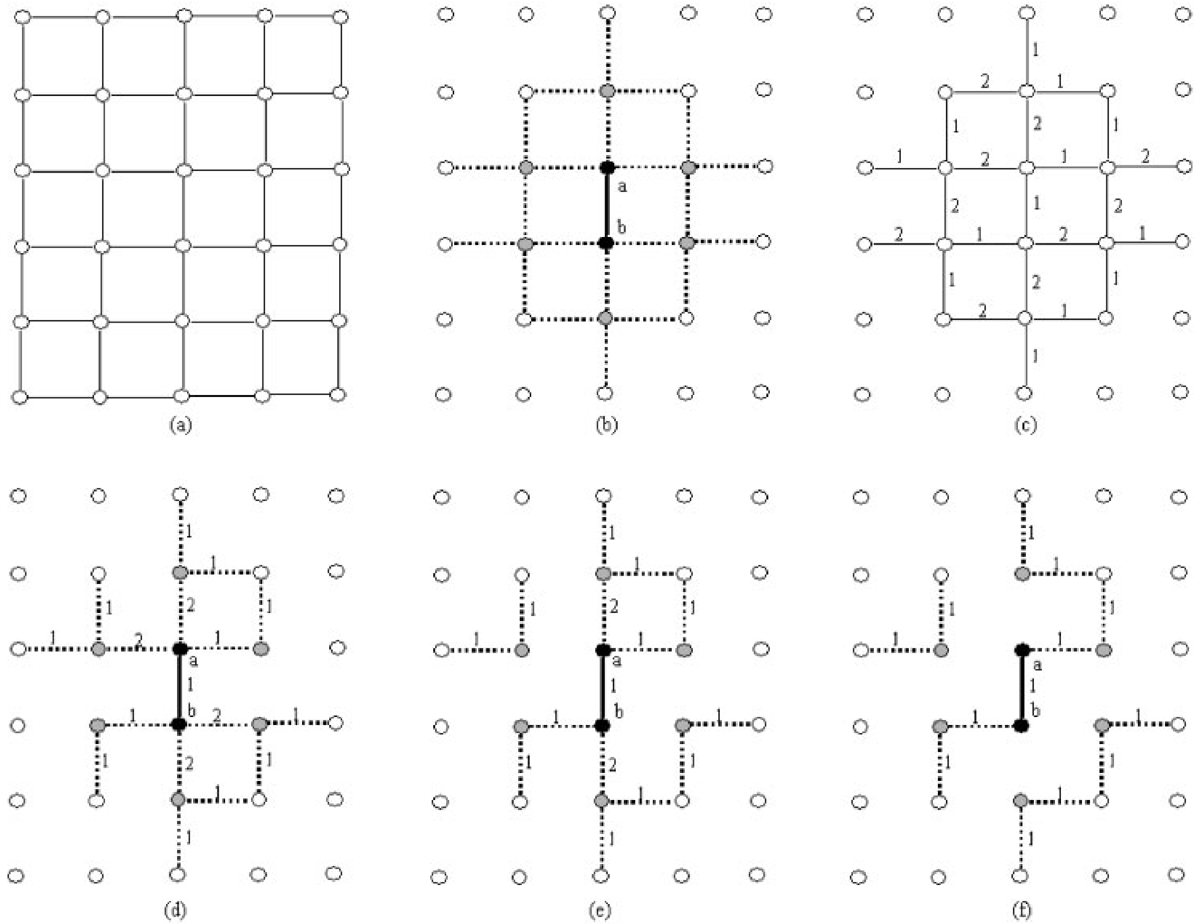

Fig. 3. (a) Reachability graph of a 5 × 6 grid; (b) Set of 22 interfering links (shown dotted) for the edge $a \leftrightarrow b$, assuming that there is only one available channel (no separate control channel). The lightly shaded nodes are neighbors of either node a or node b and must remain silent when the link $a \leftrightarrow b$ is active; (c) A partial channel allocation with 2 data channels; (d) Set of 16 edges which should remain silent when $a \leftrightarrow b$ is active on channel 1, if all nodes have one data radio. 14 of these edges (those assigned channel 1) constitute the co-channel interference set. The rest of the edges (those assigned channel 2) are forced to remain silent due to insufficient number of data radios on nodes a and b; (e) One possible configuration of 14 interfering edges which should remain silent when $a \leftrightarrow b$ is active on channel 1, if a and b each have two data radios. 12 of these edges (those assigned channel 1) constitute the co-channel interference set. The rest of the edges (those assigned channel 2) are forced to remain silent due to insufficient number of data radios on nodes a and b; (f) The set of 12 interfering edges (all due to co-channel interference) which should remain silent when $a \leftrightarrow b$ is active on channel 1, when a and b each have three data radios.

from b). We refer to this set of 22 links as the total set of interfering edges (*total interference set*) corresponding to the edge $a \leftrightarrow b$. Note that this interference is analogous to co-channel interference in cellular networks and the set of 22 links can be more aptly referred to as a *co-channel interference set*. This interpretation will be helpful in our discussion of the multichannel case.

Next, we consider the case with multiple orthogonal channels and multiple radios. At this point, we need to make a few assumptions. First, we assume that one orthogonal channel in the available spectrum is reserved as a *control channel*, and the rest, which we refer to as *data channels*, are available for data communication. Second, we envision the use of a modified MAC protocol which requires the use of the control channel for all RTS/CTS transmissions. Finally, we assume that the RTS/CTS packets contain information about the channel which will be used for data transmission, say f, so that all nodes which receive these packets can keep their radios which are tuned to channel f silent and thereby avoid a collision. Since all nodes have at least two radios (by assumption; see Section 3, item 2), we adopt a radio/channel allocation

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policy in which *one radio at each node is permanently tuned to the control channel* (*the* '*control radio*') *and the rest are available for data communication* (*the* '*data radio*(*s*)'). Consequently, channel assignment, in the context of this paper, is to be interpreted as *allocation of data channels to data radios*. Now, consider the (partial) channel allocation in Figure 3(c) when there are two available data channels. Figure 3(d) shows the total interference set of 16 edges which must remain silent when the link $a \leftrightarrow b$ is active on channel 1, assuming that all nodes have one data radio. Of these 16 edges, only those which are assigned channel 1 (it can be verified that there are 12 of them) constitute the co-channel interference set. The remaining 4 edges which are assigned channel 2 need to remain silent, not due to interference issues, but because there is only one data radio on nodes a and b. In other words, if more data radios were available at nodes a and b, it should be possible for these 4 links to be active simultaneously. In a multi-radio, multichannel framework, the total interfering set therefore captures the effect of co-channel interference as well as hardware (radio) limitations. If two data radios are available at a and b, any one of the two links incident on node a (or b) and assigned channel 2 can also be active. Figure 3(e) shows one possible configuration of the total interference set of 14 edges with two data radios, twelve of which constitute the co-channel interference set (those assigned channel 1). Finally, Figure 3(f) shows the total interference set of 12 edges when three data radios are available at a and b. Notice that, in this case, the total interference set is equal to the co-channel interference set.

We now turn to the issue of choosing an appropriate metric for static channel assignment in WMN's. But before we do so, it is interesting to note that adjacent channel constraints are not applicable to our work because of our assumption that all data channels are orthogonal. Since only three of the channels in the 2.4 GHz band are orthogonal, a better utilization of the radio spectrum in that frequency band can be achieved if our optimization models are augmented with adjacent channel separation constraints as in cellular networks. However, we do not consider this issue any further in this paper.

Typically, there will be many feasible channel assignments and we would therefore like an optimality criterion that allows us to pick one of these channel assignments. Given a set of available orthogonal data channels, the goal of a static assignment scheme should be to use the channels as 'best' as possible, thereby directly affecting the performance of a network. Some metrics which are suitable for static channel assignment are listed below. All of these attempt to increase the overall network performance by allowing more simultaneous transmissions, either directly (Problem P-1) or indirectly (Problems P-2 and P-3).

- Problem P-1: Direct maximization of the number of possible simultaneous transmissions in the network. Intuitively, such an assignment should maximize the 1-hop or link layer throughput in the network *in worst case traffic*; that is, when the traffic profile is such that there is simultaneous contending traffic on all links in the network. It is important to note that maximizing the link layer throughput through suitable channel assignment may not guarantee maximum network layer throughput (an end-to-end metric), which is a dynamic criterion and depends on the real time traffic conditions in the network. Also, the channel assignments found using this metric may not be optimal for other traffic profiles, for example, when there is only a few (source, destination) pairs, and corresponding routing paths, in the network.
- Problem P-2: Minimization of the average size (cardinality) of a co-channel interference set. This metric is analogous to the 'minimization of the average transmitter power' criterion used for topology optimization in wireless networks.
- Problem P-3: Minimization of the maximum size of any co-channel interference set, which is analogous to the 'minimization of the maximum transmitter power' criterion used for topology optimization in wireless networks. For irregular networks which have only a few edges with potentially large co-channel interference sets, this might be a better optimization criterion than the metric discussed above.

In addition to the above metrics, *channel diversity*, defined as the difference between the maximum (MAXUSAGE) and minimum (MINUSAGE) number of times any channel is used,

$$\text{channel diversity} = \text{MAXUSAGE} - \text{MINUSAGE} \tag{4}$$

is an important criterion for channel assignment. However, simply ensuring a perfectly diverse assignment (channel diversity = 0) may not affect the simultaneous

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transmission capability of a network. We will therefore use it as a secondary criterion in conjunction with the other metrics discussed above. Note that the above definition of channel diversity is a bit counterintuitive since an assignment is in fact 'more diverse' for smaller values of the r.h.s of Equation (4).

4. Direct Maximization of the Number of Possible Simultaneous Transmissions

Prior work on fixed channel assignment in cellular networks have used a 'node model', or, in other words, channels are assigned to nodes (base stations). As mentioned previously, in this paper, we adopt a 'link model' where channels are assigned to links in the reachability graph. Given a reachability graph $G = (\mathcal{N}, \mathcal{E})$, where $\mathcal{N}$ is the set of nodes and $\mathcal{E}$ is the set of edges (radio links), our objective is to find a channel assignment which would maximize the number of bidirectional links that can be activated simultaneously, subject to interference and other constraints (discussed subsequently).

Let $\mathcal{F} = [1, 2, \ldots, F]$ denote the set of available orthogonal data channels, where $F = |\mathcal{F}|$. Also, let $\mathbf{C} = [\mathbf{C}_{ef} : 1 \leq e \leq E,\ 1 \leq f \leq F]$ denote the $E \times F$ binary channel assignment matrix corresponding to the edges in $\mathcal{E}$, such that $\mathbf{C}_{ef} = 1$ if edge e is assigned channel f. Finally, we let $\vec{X} = [X_1, X_2, \ldots, X_E]^T$ be an $E \times 1$ binary vector such that $X_e = 1$ if the edge e is part of the optimal solution.

With the above notation in place, we can formally define the objective function for the fixed channel assignment problem with multiple radios and multiple channels:

$$\text{maximize} \quad \sum_{e \in \mathcal{E}} X_e \tag{5}$$

An implicit assumption in our objective function is that all radio links are potentially traffic-carrying. Or, in other words, if $\mathcal{E}_t$ is the set of potentially traffic carrying links, we assume that $\mathcal{E}_t = \mathcal{E}$. If, however, $\mathcal{E}_t \subset \mathcal{E}$, the summation in Equation (5) can be restricted to the set $e \in \mathcal{E}_t$.

In the above form of the objective function, all edges are assumed to be of equal weight. A generalization of the above objective function can be obtained by associating a weight w_e with each edge e. With an *a priori* knowledge of the traffic patterns in the network and given a routing algorithm based on a *channel-unaware* metric such as 'minimum hop',¶¶ the weight w_e can be interpreted as the fraction of the total load offered to the network which is carried on edge e. The objective function for the weighted fixed channel assignment problem is therefore:

$$\text{maximize} \quad \sum_{e \in \mathcal{E}} w_e X_e \tag{6}$$

We now discuss the constraints under which the above maximization problem is to be solved.

The first constraint forces all edges in $\mathcal{E}$ to be assigned a channel; that is,

$$\sum_{f} \mathbf{C}_{ef} = 1;\ \forall e \in \mathcal{E} \tag{7}$$

Our second constraint ensures that the number of distinct channels assigned to any node is less than or equal to K, the number of radios at each node. Towards that end, we first define an auxiliary variable matrix $\mathbf{Y} = [\mathbf{Y}_{nf} : \forall n \in \mathcal{N},\ \forall f \in \mathcal{F}]$, $0 \leq \mathbf{Y}_{nf} \leq 1$, such that $\mathbf{Y}_{nf} = 1$ if at least one edge incident on node n has been assigned channel f. Let rows(n) denote the row indices of $\mathbf{C}$ such that n is an end node in the edges corresponding to rows(n). The number of edges incident on node n is equal to $|\text{rows}(n)|$. Inequality (8) ensures that $\mathbf{Y}_{nf}$ is equal to 1 if at least one edge incident on n is assigned channel f. Note that $\mathbf{Y}_{nf}$ is free to take on a value of 1 even if channel f is not assigned to any edge incident on n. However, because of the upper bound on the number of channels that can be assigned to node n (9) and our maximization objective, such an assignment would not adversely affect the optimal cost.

$$\mathbf{Y}_{nf} = \max\{\mathbf{C}_{kf} : k \in \text{rows}(n)\}, \quad \forall n \in \mathcal{N},\ \forall f \in \mathcal{F}$$

or equivalently,

$$\mathbf{Y}_{nf} \geq \mathbf{C}_{kf};\ k \in \text{rows}(n), \quad \forall n \in \mathcal{N},\ \forall f \in \mathcal{F} \tag{8}$$

With matrix $\mathbf{Y}$ defined as above, the constraint on the maximum number of distinct channels that can be assigned to the edges incident on any node n, or equivalently, the number of distinct channels assigned

¶¶ If the routing metric is *channel-aware*, such as the WCETT (which is a maximum channel usage) metric suggested in Reference [19], channel assignment and routing are best treated as a joint optimization problem. However, this is outside the scope of this paper.

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to n, can be expressed as:

$$\sum_f \mathbf{Y}_{nf} \leq K_n; \forall n \in \mathcal{N} \quad (9)$$

where K_n is the number of radio interfaces at node n.

Our next constraint set couples the $\{X_e\}$ variables to the $[\mathbf{C}_{ef}]$ variables and is based on the notion of *potentially interfering edges*. Given that a particular (transmitter, receiver) pair is communicating on channel f:

- All nodes within CS range of the *transmitter* will sense the channel being busy via its CS mechanism and defer transmissions on channel f.
- All nodes within CS range of the *receiver* will defer transmissions on channel f to avoid collision at the receiver. Note that the DCF mechanism in 802.11 does not directly prevent these nodes from transmitting and causing a collision, except via the RTS/CTS mechanism. Given the large distances involved in mesh networks, one must expect a relatively high possibility of such hidden nodes, and hence it is likely that RTS/CTS will in fact be employed.

Since transmitters and receivers exchange roles every time an ACK is sent, we are led to the constraint that no edge which is incident on a neighbor of either the transmitter or receiver (which intend to communicate, say, on channel f) can be simultaneously active on channel f. For any bidirected edge $e = (i \leftrightarrow j) \in \mathcal{E}$, the set of its potentially interfering edges, denoted by $IE(e)$, is therefore:

$$\begin{aligned} IE(e) = \ & \text{all edges incident on } \{ne(i) \setminus j\} \ \cup \\ & \text{all edges incident on } \{(ne(j) \setminus i)\} \end{aligned} \quad (10)$$

where $ne(i)$ is the set of neighbors of node i and '$\cup$' denotes the *union* operator. Notice that $IE(e)$ is essentially the total interference set discussed in the previous section for the single radio, single channel case.

Given an edge set $\mathcal{E}$, we define an $E \times E$ *link interference matrix*, $\mathbf{LIM}$, such that the (a, b)th $(a \neq b)$ element of $\mathbf{LIM}$ is equal to 1 if (e_a, e_b) is a potentially interfering pair. All diagonal elements of $\mathbf{LIM}$ are equal to 0. Note that row (column) a of the matrix $\mathbf{LIM}$ refers to the edge e_a and the matrix $\mathbf{LIM}$

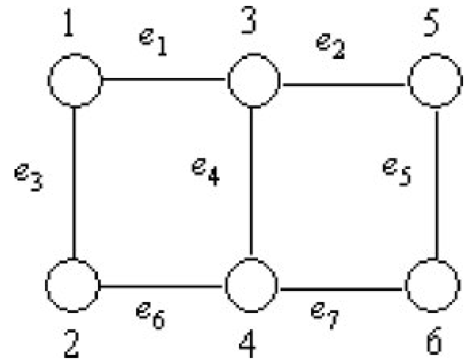

Fig. 4. A 6-node grid topology. The edges are numbered e_1 to e_7.

is symmetric.

$$\mathbf{LIM}_{ab} = \begin{cases} 1, & \text{if } e_b \in IE(e_a) \\ 0, & \text{otherwise} \end{cases} \quad (11)$$

Also, it is interesting to note that the $\mathbf{LIM}$ matrix is essentially the adjacency matrix of the *interference graph*. Given a reachability graph $G = (\mathcal{N}, \mathcal{E})$ and the $\mathbf{LIM}$ matrix, the interference graph, $I(G)$, is a graph whose node set is the edge set of G and two nodes are connected by an edge in $I(G)$ if the corresponding elements in $\mathbf{LIM}$ are equal to 1. Specifically, the nodes e_a and e_b $(e_a, e_b \in \mathcal{E})$ in $I(G)$ are joined by an edge if $\mathbf{LIM}_{ab} = \mathbf{LIM}_{ba} = 1$. See Figure 5 for an illustration.

For single radio, single channel networks, interference issues would prevent any of the edges in $IE(e_a)$ from coexisting with e_a. However, for multi-radio, multichannel networks, this condition is modified such that 'no edge in $IE(e_a)$ can coexist with e_a if they are

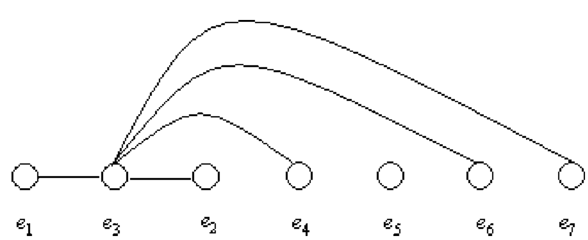

Fig. 5. A partial interference graph corresponding to the reachability graph in Figure 4. The nodes are numbered e_1 to e_7, corresponding to the edges in Figure 4. Only the edges which could potentially interfere with e_3 are shown. For example, nodes e_3 and e_1 share an edge since $\mathbf{LIM}_{31} = \mathbf{LIM}_{13} = 1$. Nodes e_3 and e_5 do not share an edge since $\mathbf{LIM}_{35} = \mathbf{LIM}_{53} = 0$.

assigned the same channel'.

$$\forall(e_a, e_b) \text{ such that } \mathbf{LIM}_{ab} = 1,$$
$$X_a + X_b \leq 1, \text{ if } \mathbf{C}_{a,:} = \mathbf{C}_{b,:} \qquad (12)$$

It is not hard to see that Equation (12) would lead to duplicate constraints for each interfering pair of edges since the matrix **LIM** is symmetric. To avoid these duplicate constraints, we rewrite Equation (12) using only the upper triangular portion of **LIM** above the leading diagonal (the diagonal elements are all 0):

$$\forall(e_a, e_b) \text{ such that } \mathrm{UT}(\mathbf{LIM})_{ab} = 1,$$
$$X_a + X_b \leq 1, \text{ if } \mathbf{C}_{a,:} = \mathbf{C}_{b,:} \qquad (13)$$

where UT(**LIM**) is the upper triangular portion of **LIM** above the leading diagonal.

Next, we note that the conditional

$$X_a + X_b \leq 1 \text{ if } \mathbf{C}_{a,:} = \mathbf{C}_{b,:}$$

can be expressed as the following system of linear inequalities:

$$X_a + X_b \leq 3 - (\mathbf{C}_{af} + \mathbf{C}_{bf}); 1 \leq f \leq F \qquad (14)$$

where F is the number of available frequency channels. Equation (14) represents a system of F equations for each (e_a, e_b) pair such that $\mathrm{UT}(\mathbf{LIM})_{ab} = 1$. It is easy to verify that if edges e_a and e_b are assigned the same channel, say f, one of the inequalities in Equation (14) will reduce to $X_a + X_b \leq 1$ (the dominating inequality) since $\mathbf{C}_{af} = \mathbf{C}_{bf} = 1$. If they are assigned different channels, say f_1 and f_2, the dominating inequality in Equation (14) will be of the form $X_a + X_b \leq 2$, meaning that both e_a and e_b are free to be chosen in the optimal solution.

Our next set of constraints are *valid inequalities*, that is, they help to improve the LP optimum obtained by relaxing the binary variables, without affecting the set of feasible integer solutions. For any node n, let $\{X_e : n \in en(e), e \in \mathcal{E}\}$ be the set of edges, one of whose end nodes is n. The notation $en(e)$ denotes the end nodes of e. Clearly, the number of edges from the set $\{X_e : n \in en(e), e \in \mathcal{E}\}$ that can be chosen in the optimal solution is limited by the number of distinct channels allocated to node n.

$$\sum_{\substack{e \in \mathcal{E} \\ n \in en(e)}} X_e - \sum_{f} \mathbf{Y}_{nf} \leq 0; \quad \forall n \in \mathcal{N} \qquad (15)$$

Our final set of constraints are also valid inequalities and is based on the notion of *potentially interfering cliques of edges*. Given an edge e, its interfering clique of edges, denoted by $IC(e)$, is a maximum cardinality subset of $IE(e)$, possibly non-unique, with the property that all edge pairs in the group $\{e \cup IC(e)\}$ are mutually interfering, if assigned the same channel. That is, for any two edges in the group $\{e \cup IC(e)\}$, say e_a and e_b, $\mathbf{LIM}_{ab} = \mathbf{LIM}_{ba} = 1$. From a graph theoretic viewpoint, the line graph corresponding to the edge set $\{e \cup IC(e)\}$ will be fully connected if all edges in $\{e \cup IC(e)\}$ are assigned the same channel.

Since all edge pairs in the set $\{e \cup IC(e)\}$ are mutually interfering if assigned the same channel, Equation (14) ensures that different channels are assigned to all edges activated from the set, up to a maximum of F. For a network with F channels, we therefore have:

$$X_e + \sum_{k \in IC(e)} X_k \leq F; \quad 1 \leq e \leq E \qquad (16)$$

A discussion on the effectiveness of the above inequalities w.r.t F and K and its impact on solver times can be found in Reference [12]. A heuristic algorithm for finding the cliques $\{e \cup IC(e) : \forall e \in \mathcal{E}\}$ is also provided in Reference [12].

4.1. Incorporating Channel Diversity

Depending on the mesh topology, simply maximizing the number of simultaneous transmissions may not generate an assignment which uses all the channels almost evenly. Figure 6(a) shows the optimal channel assignments for a 4×4 grid with two data radios on all nodes and four orthogonal data channels. All w_e's are equal to 1. The optimal assignment allows 14 of the 24 edges to transmit at the same time. Observe that 12 of the 14 edges are peripheral, which is not unexpected since we have treated all edges to be of equal weight and the core edges have a larger potential interference domain than the peripheral edges. If we now count the number of times each channel is used, we see that channel 4 is used 14 times in the assignment while channel 2 is used only 3 times. With such an uneven usage pattern, the probability of finding a channel diverse path for a randomly chosen (source, destination) node pair is reduced considerably as the length of the shortest path (in terms of number of hops) between them increases.

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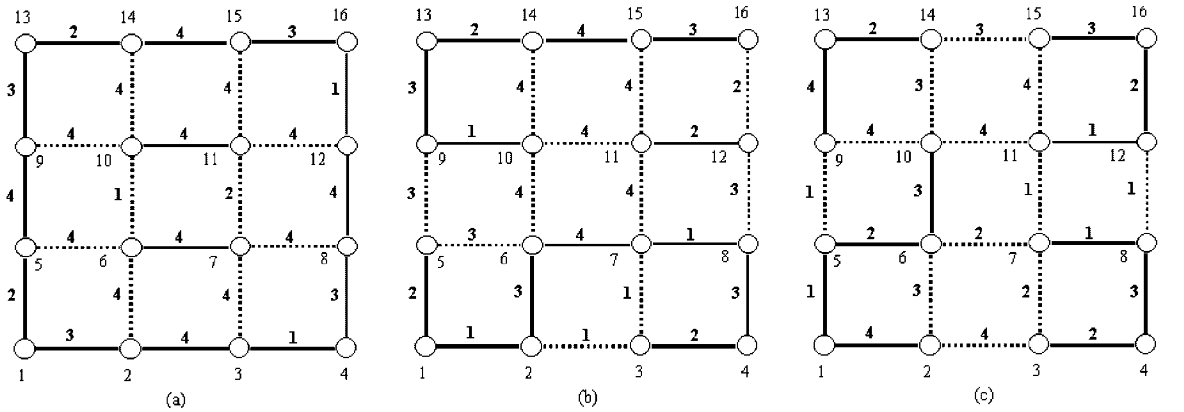

Fig. 6. Optimal channel assignments in a 4×4 grid, assuming all nodes have two data radios and four orthogonal data channels are available. All w_e's are taken to be equal to 1. (a) $\beta = 0$: 14 edges are active simultaneously, but channel usage is poor; MAXUSAGE − MINUSAGE = 11. (b) $\beta = 0.1$: 13 edges are active simultaneously, but channel usage is much better; MAXUSAGE − MINUSAGE = 2. (c) $\beta = 0.9999$: 12 edges are active simultaneously, but channel utilization is perfect; MAXUSAGE − MINUSAGE = 0.

The preceding paragraph motivates the use of a weighted objective function involving channel diversity (4) and maximum simultaneous transmission capability, as shown below:

$$\text{maximize } (1-\beta)\cdot\sum_{e\in\mathcal{E}} w_e X_e - \beta\cdot[\text{MAXUSAGE} - \text{MINUSAGE}] \quad (17)$$

where $\beta \in [0, 1]$ is a weighting factor. The variables MAXUSAGE and MAXUSAGE denote the maximum and minimum number of times any channel is used in the assignment:

$$\text{MAXUSAGE} = \max_f \sum_e \mathbf{C}_{ef} \quad (18)$$

$$\text{MINUSAGE} = \min_f \sum_e \mathbf{C}_{ef} \quad (19)$$

or equivalently,

$$\text{MAXUSAGE} - \sum_e \mathbf{C}_{ef} \geq 0; \quad \forall f \in \mathcal{F} \quad (20)$$

$$\text{MINUSAGE} - \sum_e \mathbf{C}_{ef} \leq 0; \quad \forall f \in \mathcal{F} \quad (21)$$

Figure 7 summarizes the composite ILP model for the weighted optimization problem. The number of binary variables in this model is $E(F+1)$ and

$$\text{maximize} \quad (1-\beta)\sum_e w_e X_e - \beta\cdot[MAXUSAGE - MINUSAGE]$$

subject to

$$\begin{aligned}
& 0 \leq \mathbf{Y}_{nf} \leq 1;\ \forall n \in \mathcal{N}, \forall f \in \mathcal{F} \\
& \mathbf{C}_{ef} \in \{0,1\};\ \forall e \in \mathcal{E}, \forall f \in \mathcal{F} \\
& X_e \in \{0,1\};\ \forall e \in \mathcal{E} \\
& MAXUSAGE \geq 0; \\
& MINUSAGE \geq 0; \\
& \sum_f \mathbf{C}_{ef} = 1;\ \forall e \in \mathcal{E} \\
& \mathbf{C}_{kf} - \mathbf{Y}_{nf} \leq 0;\ k \in rows(n),\ \forall n \in \mathcal{N}, \forall f \in \mathcal{F} \\
& \sum_f \mathbf{Y}_{nf} \leq K_n;\ \forall n \in \mathcal{N} \\
& X_a + X_b + (\mathbf{C}_{af} + \mathbf{C}_{bf}) \leq 3;\ \forall f \in \mathcal{F}, \forall (e_a, e_b) \text{ such that } UT(\mathbf{LIM})_{ab} = 1 \\
& \sum_{\substack{e\in\mathcal{E} \\ n \in en(e)}} X_e - \sum_{f=1}^{F} \mathbf{Y}_{nf} \leq 0;\ \forall n \in \mathcal{N} \\
& X_e + \sum_{k \in IC(e)} X_k \leq F;\ \forall e \in \mathcal{E} \\
& MAXUSAGE - \sum_e \mathbf{C}_{ef} \geq 0;\ \forall f \in \mathcal{F} \\
& MINUSAGE - \sum_e \mathbf{C}_{ef} \leq 0;\ \forall f \in \mathcal{F}
\end{aligned}$$

Fig. 7. ILP model for Problem P-1 with channel diversity. The notation UT(**LIM**) represents the upper triangular portion of the matrix **LIM**, excluding the leading diagonal. The notation rows(n) denotes the row indices of **C** such that n is an end node in the edges corresponding to rows(n).

the number of continuous variables is $NF + 2$. The number of constraints is equal to $2(E + N + F) + F\sum_n \deg_n + \sum_{a,b} \mathrm{UT}(\mathbf{LIM})_{ab}$, where $\deg_n$ is the degree of node n.

Figure 6(b) and (c) show the channel assignments for $\beta = 0.1$ and $\beta = 0.9999$. For these values of β, the number of simultaneous transmissions are 13 and 12 respectively, just 1 and 2 less than the maximum possible corresponding to $\beta = 0$. However, in both cases, channel utilization is improved significantly; for $\beta = 0.1$, MAXUSAGE − MINUSAGE = 2 and for $\beta = 0.9999$, MAXUSAGE − MINUSAGE = 0 (perfect channel diversity). Arguably, the assignment in Figure 6(b) appears to be the best choice both in terms of the number of simultaneous transmissions and channel diversity.

5. Minimization of the Average and Maximum Size of a Co-Channel Interference Set

In this section, we first consider minimization of the average size of a co-channel interference set (Problem P-2). Subsequently, we extend it to the case when the maximum (or bottleneck) size of a co-channel interference set is to be minimized (Problem P-3). As discussed previously, both these objectives are indirect metrics which are intended to improve the utilization of a network by shrinking the interference domains of the nodes, thereby allowing more simultaneous transmissions to occur. It is important to note that minimizing the average size may not minimize the maximum size, or *vice versa*.

Let $\mathbf{C} = [\mathbf{C}_{ef} : 1 \le e \le E, 1 \le f \le F]$ denote the channel assignment matrix and $\mathbf{Y} = [\mathbf{Y}_{nf} : 1 \le n \le N, 1 \le f \le F]$ be a set of auxiliary variables as in Section 5. The $\mathbf{Y}_{nf}$ variables (which are related to the $\mathbf{C}_{ef}$ variables by (8)) are used to ensure that the number of distinct channels assigned to node n is less than or equal to the number of radios on n (9). In addition, we introduce the variables $\mathbf{Z} = [\mathbf{Z}_{ef} : 1 \le e \le E, 1 \le f \le F]$, $\mathbf{Z}_{ef} \ge 0$, with $\mathbf{Z}_{ef}$ defined as follows:

$$\mathbf{Z}_{ef} = \begin{cases} \sum_{k \in IE(e)} \mathbf{C}_{kf}, & \text{if } \mathbf{C}_{ef} = 1 \\ 0, & \text{if } \mathbf{C}_{ef} = 0 \end{cases} \tag{22}$$

According to the above definition, if edge e is assigned channel f, $\mathbf{Z}_{ef}$ represents the number of potentially interfering edges of e which are assigned the same channel as e. Since each edge is assigned exactly one channel (as in Section 4; see Equation (7)), the size of the co-channel interference set of e is equal to $\sum_f \mathbf{Z}_{ef}$. The objective function for Problem P-2 can therefore be written as:

$$\text{minimize } \frac{1}{E}\sum_e \sum_f \mathbf{Z}_{ef} \equiv \sum_e \sum_f \mathbf{Z}_{ef} \tag{23}$$

If channel diversity is desired, we can use a weighted objective function:

$$\begin{aligned} &\text{minimize } (1-\beta)\cdot \sum_e \sum_f \mathbf{Z}_{ef} \\ &\quad + \beta \cdot [\text{MAXUSAGE} - \text{MINUSAGE}] \end{aligned} \tag{24}$$

where MAXUSAGE and MINUSAGE are defined as in Equations (20) and (21). Note the sign change in Equation (24) compared to Equation (17), which is reflective of the minimization objective in Problem P-2.

Next, we rewrite the conditional (22) as a linear inequality. Towards that end, first note that the equality in Equation (22) can be replaced by a '$\ge$' inequality, given the minimization of the objective function. Equation (22) can therefore be relaxed to:

$$\mathbf{Z}_{ef} \ge \begin{cases} \sum_{k \in IE(e)} \mathbf{C}_{kf}, & \text{if } \mathbf{C}_{ef} = 1 \\ 0, & \text{if } \mathbf{C}_{ef} = 0 \end{cases} \tag{25}$$

Let $\{M_e : \forall e \in \mathcal{E}\}$ be a set of constants such that (26) is satisfied.

$$M_e < -|IE(e)| = -\sum_{p=1}^{E} \mathbf{LIM}_{ep} \tag{26}$$

Note that the quantity $|IE(e)|$ can be determined from the $\mathbf{LIM}$ matrix (11); for any edge e, $|IE(e)|$ is equal to the sum of the eth row of $\mathbf{LIM}$. Then, the linear inequality (27) is equivalent to the conditional (25).

$$\mathbf{Z}_{ef} - \sum_{k \in IE(e)} \mathbf{C}_{kf} + M_e \mathbf{C}_{ef} \ge M_e \tag{27}$$

To see why, let us first consider the case when the edge e is assigned channel f, that is, $\mathbf{C}_{ef} = 1$. It is easy to verify that, in this case, Equation (27) reduces to (25). If $\mathbf{C}_{ef} = 0$, Equation (27) reduces to:

$$\mathbf{Z}_{ef} \ge M_e + \sum_{k \in IE(e)} \mathbf{C}_{kf} \tag{28}$$

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which is redundant (i.e., superseded by the non-negativity constraints on $\mathbf{Z}_{ef}$ variables, $\mathbf{Z}_{ef} \geq 0$) since the expression on the r.h.s of the above inequality is strictly negative by Equation (26).

Our final set of constraints are valid inequalities and are similar in spirit to Equation (16). Recall from our discussion in Section 4 that, for any edge e, its interfering clique of edges, $IC(e)$, is characterized by the fact that all edges in the set $\{e \cup IC(e)\}$ are mutually interfering, if all are assigned the same channel. In that case, the line graph corresponding to the set $\{e \cup IC(e)\}$ will be fully connected $\Rightarrow \sum_f \mathbf{Z}_{kf} = |IC(e)|$, $\forall k \in \{e \cup IC(e)\}$. Since an identical channel assignment for all edges in $\{e \cup IC(e)\}$ is the worst possible assignment, we have the following upper bound:

$$\begin{aligned}\sum_f \mathbf{Z}_{ef} + \sum_{k \in IC(e)} \sum_f \mathbf{Z}_{ef} &\leq |IC(e)| \cdot |e \cup IC(e)|; \quad \forall e \in \mathcal{E} \\ &= |IC(e)| \cdot (|IC(e)| + 1); \quad \forall e \in \mathcal{E} \qquad (29)\end{aligned}$$

However, these upper bounds are generally not helpful in tightening the LP relaxation of an ILP program with a minimization objective. What would be useful is a good lower bound, LB_e, which holds irrespective of the channel assignment. As illustrated and explained in Figure 8, that bound is given by:

$$\begin{aligned}LB_e = &\left\lfloor \frac{|e \cup IC(e)|}{F} \right\rfloor \\ &\times \left(|e \cup IC(e)| - \left\lfloor \frac{|e \cup IC(e)|}{F} \right\rfloor F \right) \\ &+ \frac{\left\lfloor \frac{|e \cup IC(e)|}{F} \right\rfloor \left(\left\lfloor \frac{|e \cup IC(e)|}{F} \right\rfloor - 1 \right)}{2} \times F \qquad (30)\end{aligned}$$

which leads to the following valid inequality:

$$\sum_f \mathbf{Z}_{ef} + \sum_{k \in IC(e)} \sum_f \mathbf{Z}_{ef} \geq LB_e; \quad \forall e \in \mathcal{E} \qquad (31)$$

The benefit of Equation (31) can be gauged from the fact that the LP optimum of the model without the valid inequality, when applied to a 4×4 grid (See Figure 10 for other parameters and assignments), is equal to 0, while the IP optimum, without the $1/E$ scaling in Equation (23), is 48. With the valid inequality, the LP optimum (without the $1/E$ scaling) is 16 and the integrality gap is 300%. We are currently working on

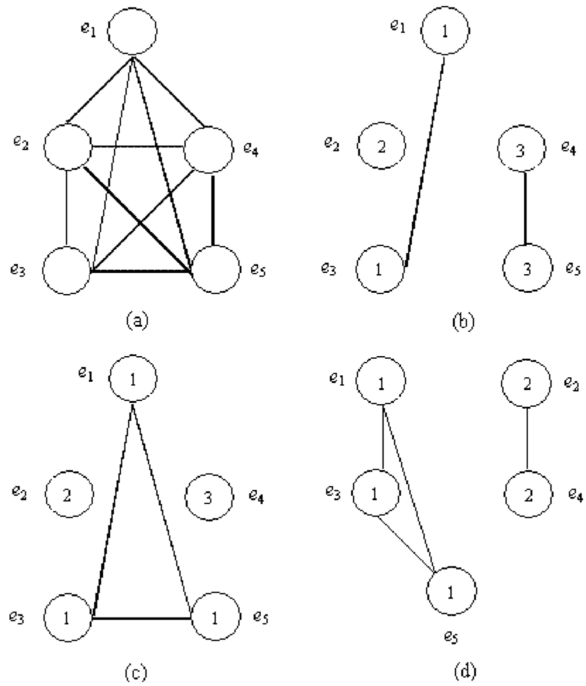

Fig. 8. (a) For any edge e_1, suppose that $IC(e_1) = \{e_2, e_3, e_4, e_5\}$. Denote $\mathcal{S} = \{e_1, e_2, e_3, e_4, e_5\}$. The line graph corresponding to the set $\mathcal{S}$ is fully connected if all nodes (we use the term 'nodes' since we are dealing with line graph representations) in the set are assigned the same channel $\Rightarrow \sum_f \mathbf{Z}_{ef} = 4, \forall e \in \mathcal{S}$; (b) Now, suppose we have $F = 3$ channels, which we assign arbitrarily to any 3 nodes from $\mathcal{S}$. The rest of the nodes have to reuse the channels. Let us call this group of nodes the *reuse group*. In this figure, one such assignment is shown. For this assignment, $\sum_{e \in \mathcal{S}} \sum_f \mathbf{Z}_{ef} = 2\,(|\mathcal{S}| - F) = 4$, where the factor 2 is due to the fact that a common channel assignment affects the interference set of both its end nodes. In fact, it is not hard to convince oneself that the best possible (in terms of the sizes of the co-channel interference sets) assignment for the group of nodes in $\mathcal{S}$ is one in which the reuse group uses a channel exactly once; (c) In this assignment, the reuse group uses a channel more than once and $\sum_{e \in \mathcal{S}} \sum_f \mathbf{Z}_{ef}$ is equal to 6; (d) In general, depending on F and $|\mathcal{S}|$, we may have more than one reuse group, as shown here for $F = 2$. Moreover, it is possible for a reuse group to be partially full (i.e., the number of nodes in the reuse group $<F$); for example, the singleton set $\{e_5\}$ in our example. A full reuse group in (d) is the set $\{e_3, e_4\}$. It can be shown that the best channel assignment, in general, is one which uses each channel exactly once within each (full or partial) reuse group. In this case, $\sum_{e \in \mathcal{S}} \sum_f \mathbf{Z}_{ef} = x\,(|\mathcal{S}| - xF) + \frac{x(x-1)}{2} \cdot F$ where x is the number of full groups $= \left\lfloor \frac{S}{F} \right\rfloor$ and $(|\mathcal{S}| - xF)$ is the number of nodes in the partially full reuse group.

identifying other valid inequalities which can be used to tighten the LP relaxation further.

This completes our discussion of the ILP model for Problem P-2. A summary of the formulation is provided in Figure 9. The number of binary variables in this

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$$\text{minimize} \quad (1-\beta)\sum_e \sum_f \mathbf{Z}_{ef} + \beta \cdot [MAXUSAGE - MINUSAGE]$$

subject to

$$\begin{aligned}
0 \le \mathbf{Y}_{nf} &\le 1;\ \forall n \in \mathcal{N}, \forall f \in \mathcal{F}\\
\mathbf{C}_{ef} &\in \{0,1\};\ \forall e \in \mathcal{E}, \forall f \in \mathcal{F}\\
\mathbf{Z}_{ef} &\ge 0;\ \forall e \in \mathcal{E}, \forall f \in \mathcal{F}\\
MAXUSAGE &\ge 0;\\
MINUSAGE &\ge 0;\\
\sum_f \mathbf{C}_{ef} &= 1;\ \forall e \in \mathcal{E}\\
\mathbf{C}_{kf} - \mathbf{Y}_{nf} &\le 0;\ k \in rows(n),\\
&\qquad \forall n \in \mathcal{N}, \forall f \in \mathcal{F}\\
\sum_f \mathbf{Y}_{nf} &\le K_n;\ \forall n \in \mathcal{N}\\
\mathbf{Z}_{ef} - \sum_{k \in IE(e)} \mathbf{C}_{kf} + M_e \mathbf{C}_{ef} &\ge M_e\ \forall e \in \mathcal{E}, \forall f \in \mathcal{F}\\
\sum_f \mathbf{Z}_{ef} + \sum_{k \in IC(e)} \sum_f \mathbf{Z}_{kf} &\ge LB_e;\ \forall e \in \mathcal{E}\\
MAXUSAGE - \sum_e \mathbf{C}_{ef} &\ge 0;\ \forall f \in \mathcal{F}\\
MINUSAGE - \sum_e \mathbf{C}_{ef} &\le 0;\ \forall f \in \mathcal{F}
\end{aligned}$$

Fig. 9. ILP model for Problem P-2 with channel diversity. The same model can be used for Problem P-3 if the objective function (33) is used and constraint (32) is added to the above. The constants $\{M_e : \forall e \in \mathcal{E}\}$ and $\{LB_e : \forall e \in \mathcal{E}\}$ are computed according to Equations (26) and (30) respectively.

model is EF and the number of continuous variables is $(N+E)F+2$. The number of constraints is equal to $(N+E+2)(F+1)+F\sum_n \deg_n$.

5.1. Minimization of the Maximum Size of a Co-Channel Interference Set

The ILP formulation for Problem P-2 can be easily extended if minimization of the maximum size of a co-channel interference set is the objective. To do so, we first define the scalar variable t defined as follows:

$$t = \max_{e,f}(\mathbf{Z}_{ef})$$

or equivalently,

$$t - \mathbf{Z}_{ef} \ge 0; \quad \forall e \in \mathcal{E},\ \forall f \in \mathcal{F} \tag{32}$$

The objective function for Problem P-3, weighted with channel diversity, is:

$$\begin{aligned}\text{minimize } & (1-\beta)\cdot t\\ & + \beta \cdot [\text{MAXUSAGE} - \text{MINUSAGE}]\end{aligned} \tag{33}$$

All other constraints are identical to Problem P-2. In Figure 10, we provide simulation results for a 4×4 grid and compare the sizes of the interference regions for both the single channel, single radio case and the multichannel, multi-radio case for $F = 4$ and $K_n = 2, \forall n$. For this example, the average interference domain size is 2 for both P-2 and P-3, which is purely coincidental (see Figure 11). The maximum interference domain size, on the other hand, is 4 for P-2 and 2 for P-3. This shows that minimizing the average interference domain size may not minimize the maximum interference domain size.

In Figure 11, we show the optimal channel assignments considering metrics P-2 and P-3 for $F = 2$ and $K_n = 2, \forall n$. In this case, the average interference domain size is 5 for P-2, compared to 5.67 for P-3. This shows that minimizing the maximum interference domain size may not minimize the average interference domain size. The maximum interference domain size, on the other hand, is 8 for P-2 and 7 for P-3.

6. Simulation Results

In this section, we present our simulation results for the channel assignments schemes presented in the previous sections. The aggregate 1-hop throughput as a function of the CS range is investigated for different policies. All simulations were conducted using the OPNET network simulator. The parameters which we have used for our simulations are listed below:

- Number of nodes = 16, arranged as a regular (i.e., the physical distance between any two neighbors is the same) 2-D 4×4 grid.
- Grid separation distance $d = 10$ m.
- All mesh nodes are routers.
- Each node is equipped with two 802.11a radios.
- Number of orthogonal data channels = 4.
- Transmission range = 10 m.
- A flow refers to an IP connection between a source destination pair.
- Packet size = 1500 bytes, sendingrate = 12 Mbps packets/second.
- Transmission power = 1 mW.
- CS threshold = −95 dB, which is equivalent to a CS range of 261 m.
- RTS/CTS mechanism is disabled.
- Routing is done by a static routing table built in each router to control the active flows.

Figures 12 and 13 show the aggregate 1-hop throughput, as a function of the CS range for a 4×4

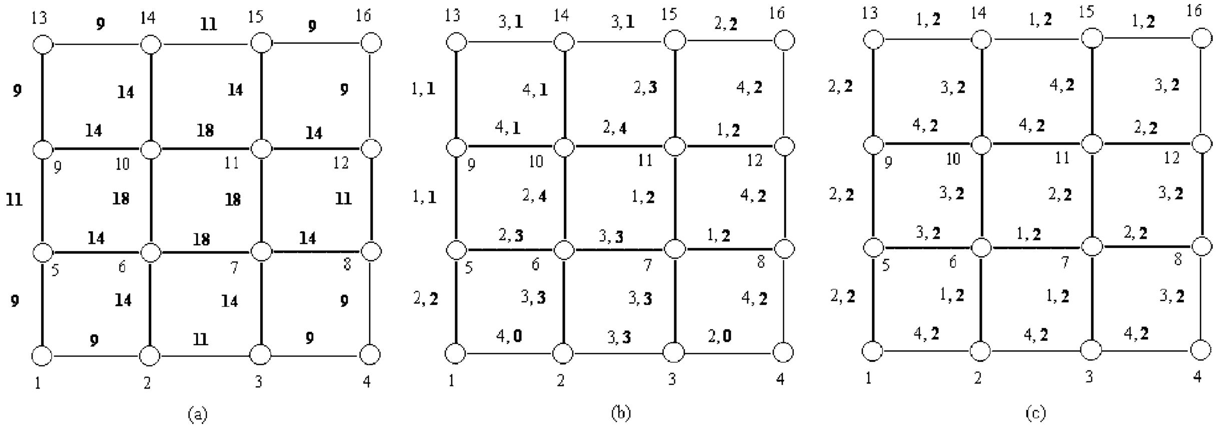

Fig. 10. Illustrating how multiple channels and multiple radios can reduce the average and maximum size of co-channel interference sets. (a) The labels (shown boldfaced) on the edges represent the size of their interference regions, for *single channel* and *single radio*. For example, the label 18 on the edge $6 \leftrightarrow 7$ means it forces 18 other edges to remain silent when it is active; (b) Optimal channel assignments when the average size of a co-channel interference set is minimized ($\beta = 0$, Problem P-2) for $K_n = 2, \forall n$ and $F = 4$. The labels on the edges have 2 parameters. The first parameter represents the channel assigned to the edge and the second parameter represents the size of its co-channel interference set. For example, the label (2,4) on the edge $10 \leftrightarrow 11$ means that it is assigned channel 2 and the size of its co-channel interference set is 4. Observe that this is the maximum size of any co-channel interference set, compared to 18 in (a). Note also that the assignment is almost perfectly diverse since MAXUSAGE = 7 (corresponding to channel 2) and MINUSAGE = 5 (corresponding to channel 1); (c) Optimal channel assignments when the maximum size of a co-channel interference set is minimized ($\beta = 0$, Problem P-3) for $K_n = 2, \forall n$ and $F = 4$. As in (b), the labels on the edges have 2 parameters; the first parameter represents the channel assigned to the edge and the second parameter represents the size of its co-channel interference set. The maximum size of any co-channel interference set is now 2 (in fact, it is the same for all edges), compared to 4 in (b). Even though the average size of a co-channel interference set is identical for both (b) and (c), it is purely coincidental. The assignment is perfectly diverse since each channel is used 6 times, which is also coincidental.

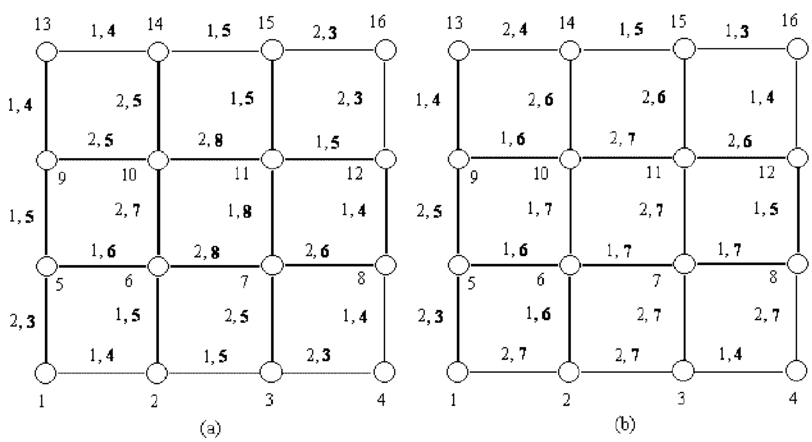

Fig. 11. (a) Optimal channel assignments when the average size of a co-channel interference set is minimized ($\beta = 0$, Problem P-2) for $F = 2$ and $K_n = 2, \forall n$. As in Figure 10, The labels on the edges have 2 parameters. The first parameter represents the channel assigned to the edge and the second parameter represents the size of its co-channel interference set. The average interference domain size is 5 and the maximum interference domain size is 8. The assignment is almost perfectly diverse since MAXUSAGE = 13 (corresponding to channel 1) and MINUSAGE = 11 (corresponding to channel 2); (b) Optimal channel assignments when the maximum size of a co-channel interference set is minimized ($\beta = 0$, Problem P-3) for $K_n = 2, \forall n$ and $F = 2$. The average interference domain size is 5.67 and the maximum interference domain size is 7, compared to 8 in (a). The assignment is perfectly diverse since each channel is used 12 times.

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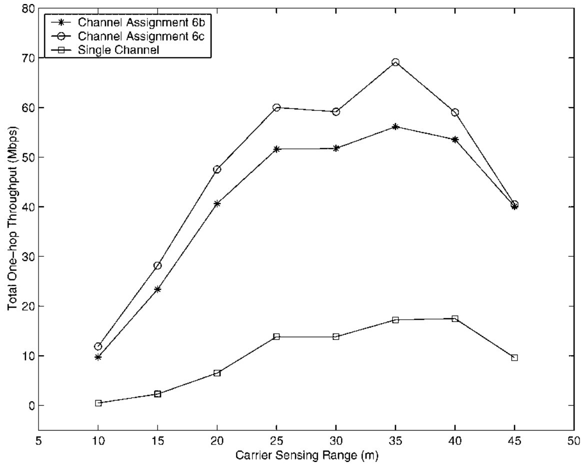

Fig. 12. Total total end-to-end throughput (in Mbps) for channel assignment presented in Figure 6, as a function of the carrier sensing range in a 4 × 4 802.11b mesh grid.

802.11b mesh network with channel assignments given earlier in Figures 6 and 10, respectively.

The enhancement in the end-to-end throughput is clearly evident when multiple radios are available. This is a consequence of improved channel reuse which is possible with multiple radios. The implication of CS range is that any source within this range of the reference transmitter will sense the ongoing transmission and defer its own. Moreover, when the carrier sensing range is small, the hidden terminal problem appears to be significantly alleviated with increasing number of radios and channels, as indicated by the jump in throughput in the aforementioned figures. This is intuitively justified since multiple radios and orthogonal channels effectively shrinks the collision domain size. Although the effectiveness of the carrier sensing mechanism in avoiding collisions is reduced if the CS range is small relative to the interference range, using multiple radios with orthogonal channels largely alleviates the collision problem, thereby leading to significantly enhanced throughput when the CS range is small but multiple channels are available.

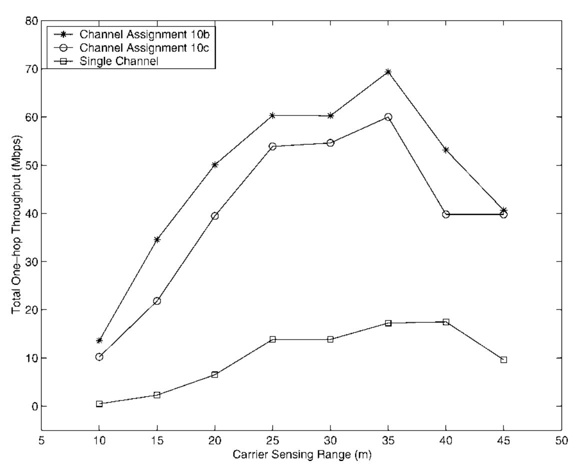

Fig. 13. Total total end-to-end throughput (in Mbps) for channel assignment presented in Figure 10, as a function of the carrier sensing range in a 4 × 4 802.11b mesh grid.

7. Conclusion

We have considered the static channel assignment problem for multi-radio, multichannel 802.11 WMN. We discussed its similarities and differences with the channel assignment problem in cellular networks and presented four metrics based on which mesh channel assignments can be obtained. Three of these metrics attempt to maximize simultaneous transmissions in a mesh network, either directly or indirectly. The fourth metric quantifies the diversity of a particular assignment and can be used as a secondary criterion to the other three metrics. Related optimization models have also been developed and the simulation results have shown clear enhancement in the end-to-end throughput due to the improved channel reuse which is a consequence of the deployment of multiple radios.

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