PMU Deployment for Optimal State Estimation Performance

Yue Yang, *Student Member IEEE*, and Sumit Roy, *Fellow IEEE*

***Abstract*—The future ‘smart’ grid will see increasing deployments of intelligent electronic devices (IED), that sense the grid state variables so as to support enhanced, real-time monitoring and control within a service area for achieving increased operational efficiency, reliability and security. Increasing penetration of Phasor Measurement Units (PMUs) like devices within the distribution grid are anticipated; however, due to the high cost of PMU installation, their deployment will continue to be selective for the foreseeable future. Previous work has largely focused on achieving full topological observability with (appropriately placed) minimal number of PMUs for a given network. We observe that there typically exist *multiple* minimal-PMU sets that achieve full observability, affording an additional degree of freedom to select an optimal choice among this set. We define the desired solution as the placement that also achieves *best state estimation* performance. Accordingly, we propose *two* approaches - a greedy algorithm and an integer programming method to determine the PMU locations that minimize the mean-squared state estimation error.**

***Index Terms*—Phasor Measurement Units (PMU), State estimation, Deployment**

I. Introduction

Monitoring and estimation of state at buses is an important component of Smart Grid operations. The most commonly used state measurement within an Energy Management System (EMS) includes measurement of line power flow and bus power injection. However, real-time monitoring of future power grid for greater reliability requires more advanced sensors that offer more frequent, higher precision measurements, that are networked to exploit the benefits from data aggregation.

Phasor Measurement Unit (PMU) is such an advanced device capable of providing synchronous measurements of the complex voltage at a bus where it is located, as well as the complex current on all lines connected to the bus. Currently, PMUs along with protective devices are mostly employed within the transmission network. In the future, we anticipate that a suitable lower-cost version will be deployed within the distribution grid. These new PMU-like IEDs will sense grid status locally, process and transmit this information to a suitable data aggregation and control center over a yet-to-be-defined communication network based on technologies described in [1].

It is evident that for a highly connected network and with limits on the number of available PMUs, careful choice of PMU locations is critical to achieving full topological observability [2][1]. This problem has attracted the most attention to date, as in [3]-[5] that propose different algorithmic solutions. In [4], the authors use integer programming method whereas the Bisecting Search Method was used in [3]; the latter tests the observability of candidate deployments at each step and improves the convergence speed of the search. In [5], the authors propose a Greedy Depth First Search Method (GDFSM) to reduce the computation complexity of the prior approach, wherein the algorithm preferentially places PMUs at buses with maximum degree iteratively. All these algorithms typically yield *several* minimal deployments that achieve full topological observability.

Clearly, we may further choose a ‘best’ among the various PMU placement solutions obtained above, by considering state estimation performance. In order to achieve this, we first propose a modified state estimation model that includes the states of *all buses* in the network, regardless of whether full observability is achieved or not. Based on this model, we define estimation performance for a given PMU deployment as the determinant of the error covariance matrix (or equivalently, the volume of the error uncertainty region). Minimizing this is our optimization target, which is known to be a non-convex problem. Therefore, we initially propose a greedy algorithm to find the optimal PMU deployment; subsequently we use an integer programming method that reformulates the PMU deployment problem into a solvable optimization problem.

The major contributions of our work may be summarized as: 1) proposed a modified state estimation model that captures all network states (both unobservable and observable buses); 2) applied the integer programming method for the PMU deployment problem to achieve the best estimation performance. We emphasize the clear methodological differences between our approach and those in relevant prior art. For example, [6] solves this problem essentially via exhaustive search, which is less efficient than the greedy approach used in [7]. However, the authors in [7] assume that the whole network is already observable by conventional meters and the purpose of PMU placement is to improve state estimation performance. In contrast, at each stage of our proposed iterative approach, we obtain the optimal PMU placement defined in terms of the minimum estimation error, irrespective of whether the whole network is fully observable[2], that is clearly distinct from [7] and [8].

The rest of the paper is organized as follows: in Sections II and III, we formulate the modified state estimation model and develop the state estimator. The greedy algorithm and integer

[1]Topological observability of a network implies that there exists at least one spanning tree of full rank [3].

[2]Clearly, given sufficient number of PMUs, our algorithm ultimately achieves full observability AND provides the optimal placement in terms of minimal estimator MSE.

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programming method are proposed and analyzed in the Section IV and Section V respectively. Section VI contains numerical results and discussion of performance, followed by concluding remarks in Section VII.

II. Preliminary

A. Network Topology

Consider a directed network topology for power grid $G = [V, E]$, where $V = \{v_1, v_2 \ldots v_{|V|}\}$ and $E = \{e_1, e_2 \ldots e_{|E|}\}$. The vertex v_i denotes the bus i. If there is a power line between bus i and bus j, then the graph contains an edge $e \in E$ between v_i and v_j. Furthermore, we arbitrarily assign the direction for each edge as that of the reference current [9].

B. PMU Deployment

A PMU deployed at a bus can measure the complex voltage at that node (vertex) and the complex current along *all* edges connected to this vertex. As a result, the complex voltage on any adjacent vertex can also be indirectly 'measured' by using Ohm's Law. Accordingly, we divide all vertices into three groups with respect to a given topology and a PMU deployment, as follows:

- Group 1 vertex set L_1: vertices allocated with PMU.
- Group 2 vertex set L_2: vertices adjacent to PMU located vertices.
- Group 3 vertex set L_3: unobservable vertices.

Since L_1 and L_2 are exclusive, we define $V' = L_1 + L_2$ to be the set of *measurable vertices*. For simplicity, we do not apply zero-injection in our work and, thus, the Group 1 vertex and Group 2 vertex are the only two kinds of observable vertices. Furthermore, since L_1, L_2, L_3 are all exclusive, then $L_1 + L_2 + L_3 = V$. The *measurable edge set* E' contains all edges between any two measurable vertices, whose direction is from PMU located vertex to its neighbor. It is noted that if two PMUs are located adjacently, then the edge between them counts twice in E' but with opposite direction. Finally, $\Psi(L_1)$ denotes the *PMU deployment* with respect to L_1.

III. System Model

Following [6], we employ the linear state estimation model given by

$$\mathbf{x}_k = U_k\,\mathbf{x}_{k-1} + \mathbf{q}_k, \tag{1}$$

$$\mathbf{z}_k = H_k\,\mathbf{x}_k + \mathbf{r}_k, \tag{2}$$

where

- $\mathbf{x}_k$: $|V| \times 1$ state vector of complex voltage at each vertex at time k.
- $\mathbf{z}_k$: $m \times 1$ measurement vector of complex voltage at the PMU located vertices and complex current along measurable edges at time k.
- U_k: $|V| \times |V|$ state transition matrix at time k.
- H_k: $m \times |V|$ observation matrix at time k, assumed time invariant (H).
- $\mathbf{q}_k, \mathbf{r}_k$: $|V| \times 1$ and $m \times 1$ vector of process and measurement noise respectively at time k, which are assumed to be time invariant, mutually independent and identical complex Gaussian random variables.

$$\mathbf{q} \sim N(0, Q), \mathbf{r} \sim N(0, R), \tag{3}$$

where the matrix Q and R represent the process and measurement noise covariance matrix.

A. Process Model

We will assume the power system to be in dynamic steady-state, i.e. the bus voltages at time $k+1$ are the same as those at time k and the process noise matrix Q approaches zero. Therefore, the state transition matrix $U_k = I$ over the observation duration. Hence, our process model reduces to

$$\mathbf{x}_k = I\,\mathbf{x}_{k-1}. \tag{4}$$

B. Measurement Model

For the measurement model in the form $\mathbf{z}_k = H\mathbf{x}_k + \mathbf{r}$, the elements in vector $\mathbf{z}_k$ can be divided into three classes: a) voltage measurements which measure the complex voltage on the PMU located vertices (L_1), b) current measurements which measure the complex current along the measurable edges (E'), and c) the pseudo-voltage measurements which represent the complex voltage on the unobservable vertices (L_3). Because these three classes of measurements are all mutually exclusive, the row dimension of vector $\mathbf{z}_k$ and matrix H is $m = |L_1| + |E'| + |L_3|$.

1) Voltage Measurements

The measurement equation can be described as

$$\mathbf{z}_k^{L_1} = I\,\mathbf{x}_k^{L_1} + \mathbf{r}^{L_1}, \tag{5}$$

where $\mathbf{z}_k^{L_1}$ and $\mathbf{x}_k^{L_1}$ denotes $|L_1| \times 1$ measurement and state sub vector of complex voltage on L_1 at time k respectively. The identity observation matrix denotes the fact that this is a direct measurement at the PMU locations, and elements of the measurement noise $\mathbf{r}^{L_1}$ are assumed to have unit variance (normalized without loss of generality).

2) Current Measurements

According to [3], the measurement equation is given as

$$\mathbf{z}_k^{E'} = \begin{bmatrix} MYD_{L_1}^T & MYD_{L_2}^T \end{bmatrix} \begin{bmatrix} \mathbf{x}_k^{L_1} \\ \mathbf{x}_k^{L_2} \end{bmatrix} + \mathbf{r}^{E'}, \tag{6}$$

where $\mathbf{z}_k^{E'}$ means $|E'| \times 1$ measurement sub vector of complex current on E' at time k. $\mathbf{x}_k^{L_2}$ means $|L_2| \times 1$ state sub vector of complex voltage on L_2 at time k. The measurement-to-edge incidence matrix with respect to the network topology and a given PMU placement, $M_{|E'| \times |E|}$, is defined by

$$[M]_{ij} = \begin{cases} 1 & \text{if measurable edge } e_i \text{ in } E' \text{ equals to edge } e_j \\ & \text{in } E \text{ and } e_i, e_j \text{ has same direction,} \\ -1 & \text{if measurable edge } e_i \text{ in } E' \text{ equals to edge } e_j \\ & \text{in } E \text{ and } e_i, e_j \text{ has opposite direction,} \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

The matrix $Y_{|E| \times |E|}$ is a diagonal matrix, whose elements are the admittance of each edge in the topology. The matrix D_{L_1}

is the Group 1 vertex-to-edge incidence matrix with dimension $|L_1| \times |E|$, which is defined by

$$[D_{L_1}]_{ij} = \begin{cases} 1 & \text{if } v_i \text{ in } L_1 \text{ is incident to edge } e_j \text{ in } E \\ & \text{and } v_i \text{ is at the head side of } e_j, \\ -1 & \text{if } v_i \text{ in } L_1 \text{ is incident to edge } e_j \text{ in } E \\ & \text{and } v_i \text{ is at the end side of } e_j, \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

The matrix D_{L_2}, the Group 2 vertex-to-edge incidence matrix, has a similar definition and property. As before, we also normalize the elements in vector $\mathbf{r}^{E'}$ to have unit variance.

3) Pseudo-voltage Measurements

Since the Group 3 vertices cannot be directly measured or indirectly calculated, we assign pseudo-voltage measurements for L_3. The measurement equation can be described as

$$\mathbf{z}_k^{L_3} = I\,\mathbf{x}_k^{L_3} + \mathbf{r}^{L_3}, \tag{9}$$

where $\mathbf{z}_k^{L_3}$ and $\mathbf{x}_k^{L_3}$ denotes $|L_3| \times 1$ pseudo-voltage measurement and state sub vector of complex voltage on L_3 at time k. The vector $\mathbf{r}^{L_3}$ elements are assumed to have large variance a_R, suggesting the precision of pseudo-measurement is low.

We combine the three classes of measurements to produce the complete state estimation model below.

$$\mathbf{x}_k = I\,\mathbf{x}_{k-1}, \tag{10}$$

$$\mathbf{z}_k = \begin{bmatrix} I & 0 & 0 \\ MYD_{L_1}^T & MYD_{L_2}^T & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} \mathbf{x}_k^{L_1} \\ \mathbf{x}_k^{L_2} \\ \mathbf{x}_k^{L_3} \end{bmatrix} + \begin{bmatrix} \mathbf{r}^{L_1} \\ \mathbf{r}^{E'} \\ \mathbf{r}^{L_3} \end{bmatrix}, \tag{11}$$

$$\mathbf{r} \sim N(0, R),\ R = \begin{bmatrix} I_{(|L_1|+|E'|)} & 0 \\ 0 & a_R \cdot I_{|L_3|} \end{bmatrix}. \tag{12}$$

In the absence of any process noise, the Weighted Least Square (WLS) state estimator under the condition where $\mathbf{r}$ follows complex Gaussian distribution is given as [10]: $\hat{\mathbf{x}} \sim N(\mathbf{x}, (H^TWH)^{-1})$, where the weighting matrix $W = R^{-1}$ and steady-state error covariance matrix $P = (H^TWH)^{-1}$. According to [11], the optimal state estimation performance corresponds to maximizing the determinant of Fisher information matrix $P' = P^{-1} = H^TWH$, which is equivalent to minimizing the uncertainty of the state estimator. In summary, the central optimization problem can be stated as follows: given an upper bound of the number of PMUs c, find the optimal PMU deployment Ψ such that state estimation performance $\Omega(\Psi) = det(P')$ is maximized.

IV. Greedy Algorithm

In this section, we propose a greedy algorithm for obtaining a solution to the PMU deployment problem stated above. Initially, the first PMU is placed on each vertex in turn and the corresponding $\Omega(\Psi)$ computed. The PMU is placed on the vertex that maximizes $\Omega(\Psi)$. Subsequently, an exhaustive search way is conducted to place the second PMU on one of the remaining vertices that maximizes $\Omega(\Psi)$ for the two-PMU case. This continues until the bound of the number of PMUs c is achieved. The algorithm details are given below:

Input:

- Network topology for the power grid $G = [V, E]$,
- The upper bound of the number of PMUs c.

Initialization:

- The current number of PMUs in the network, $n = 1$,
- The set of available vertices for PMU location when placing the n_{th} PMU, $U(n = 1) = V$,

Procedure

```
while n ≤ c do
  for v(n) ∈ U(n) do
    Construct H(Ψ) for Ψ(L1 = {v(1), v(2), ...v(n)})
    Compute Ω(Ψ)
  end for
  Place PMU on v(n), such that Ω(Ψ) is maximized.
  Update Ψ, U(n + 1) = U(n) − {v(n)} and n = n + 1
end while
```

Output: The PMU deployment $\Psi(L_1 = \{v(1), v(2), ...v(c)\})$ that maximizes $\Omega(\Psi)$.

V. Integer Programming Optimization Method

In this section, we use integer programming method to reformulate the above problem into a series of tractable optimization equations. The resulting optimizations can be solved by MATLAB toolbox to yield the optimal PMU deployment.

For a graph, the connectivity matrix $C_{|V|\times|V|}$ is defined by

$$[C]_{ij} = \begin{cases} 1 & \text{if } i = j \text{ or } v_i \text{ is adjacent to } v_j, \\ 0 & \text{otherwise.} \end{cases} \tag{13}$$

Similar to D_{L_1} and D_{L_2}, the directed vertex-to-edge incidence matrix with respect to the entire network is denoted by D. We also use $|D|$ to denote the undirected vertex-to-edge incidence matrix, whose elements are the absolute values of those in D. For convenience we also define a new operator, the *n-order matrix logical multiplication*, denoted by $\otimes^n$, as follows. For A, B, C matrices

$$[C = A \otimes^n B]_{ij} = \begin{cases} 1 & \text{if } \sum_l a_{il} b_{lj} \geq n, \\ 0 & \text{otherwise.} \end{cases} \tag{14}$$

The PMU deployment locations are captured by the binary vector $\mathbf{X}_{|V|\times 1} = [x_1, x_2, ...x_{|V|}]^T$, where

$$x_i = \begin{cases} 1 & \text{if PMU is located on vertex } v_i, \\ 0 & \text{otherwise.} \end{cases} \tag{15}$$

Then the observable vertices are those in the binary vector $\mathbf{X}_{ob} = C \otimes^1 \mathbf{X}$, that satisfies the following inequality:

$$C\mathbf{X} \cdot \frac{1}{|E|} \leq \mathbf{X}_{ob} \leq C\mathbf{X} \tag{16}$$

We also define unobservable vertices vector $\mathbf{X}_{unob} = \mathbf{1} - \mathbf{X}_{ob}$ and general observable vertices vector $\mathbf{X}_{geob} = \mathbf{X}_{unob} + \mathbf{X}$.

In the measurement model, the measurement-to-edge incidence matrix M can be divided into two sub matrices:

$$M_{(2|E|\times|E|)} = \begin{bmatrix} M^1 \\ M^2 \end{bmatrix}. \quad (17)$$

The top matrix $M^1_{|E|\times|E|} = diag(|D|^T \otimes^2 \mathbf{X}_{ob})$ relates the current measurements to all the edges in the network. Similar to (16), the matrix M^1 satisfies the following:

$$M^1 = diag(\mathbf{X}_1), \text{where } |D|^T\mathbf{X}_{ob} - \mathbf{1} \le \mathbf{X}_1 \le \frac{|D|^T\mathbf{X}_{ob}}{2} \quad (18)$$

Furthermore, if two PMUs are placed on adjacent vertices, the edge between them is measured twice. The first measurement is included in M^1, while the second one is represented in $M^2_{|E|\times|E|} = diag(|D|^T \otimes^2 \mathbf{X})$, where M^2 satisfies:

$$M^2 = diag(\mathbf{X}_2), \text{where } |D|^T\mathbf{X} - \mathbf{1} \le \mathbf{X}_2 \le \frac{|D|^T\mathbf{X}}{2} \quad (19)$$

Thereafter, if we swap the current measurements rows and pesudo-voltage measurements rows in matrix H, we obtain

$$H_{(|V|+2|E|)\times|V|} = \begin{bmatrix} diag(\mathbf{X}_{geob}) \\ MYD^T \end{bmatrix}. \quad (20)$$

The resulting measurement noise covariance matrix becomes

$$R = diag\left(\begin{bmatrix} \mathbf{1}_{|V|\times 1} + \mathbf{X}_{unob}\cdot(a_R - 1) \\ \mathbf{1}_{2|E|\times 1} \end{bmatrix}\right), \quad (21)$$

Accordingly, the weighting matrix W becomes

$$W = R^{-1} = diag\left(\begin{bmatrix} \mathbf{1}_{|V|\times 1} - \mathbf{X}_{unob}\cdot(\frac{a_R-1}{a_R}) \\ \mathbf{1}_{2|E|\times 1} \end{bmatrix}\right). \quad (22)$$

In summary, the optimization can be represented in terms of binary vectors and matrices as:

$$\mathbf{X} := arg\ max\ det(P' = H^TWH) \quad (23)$$

$$\text{s.t.} \sum_{i=1}^{|V|} x_i \le c \quad (24)$$

$$C\mathbf{X}\cdot\frac{1}{|E|} \le \mathbf{X}_{ob} \le C\mathbf{X} \text{ and } \mathbf{X}_{ob} \text{ is binary} \quad (25)$$

$$|D|^T\mathbf{X}_{ob} - \mathbf{1} \le \mathbf{X}_1 \le \frac{|D|^T\mathbf{X}_{ob}}{2} \text{ and } \mathbf{X}_1 \text{ is binary} \quad (26)$$

$$|D|^T\mathbf{X} - \mathbf{1} \le \mathbf{X}_2 \le \frac{|D|^T\mathbf{X}}{2} \text{ and } \mathbf{X}_2 \text{ is binary} \quad (27)$$

In the case of *full observability*, we require $\mathbf{X}_{ob} = \mathbf{1}$, which means $C\mathbf{X} \ge \mathbf{1}$. Then, according to (18), the matrix M^1 is simplified into identity matrix. Furthermore, because $\mathbf{X}_{unob} = \mathbf{0}$, then $\mathbf{X}_{geob} = \mathbf{X}$. Accordingly, the matrices R and W are identity and the optimization simplifies to

$$\mathbf{X} := arg\ max\ det(P' = H^TWH) \quad (28)$$

$$\text{s.t.} \sum_{i=1}^{|V|} x_i \le c \quad (29)$$

$$C\mathbf{X} \ge \mathbf{1} \quad (30)$$

$$|D|^T\mathbf{X} - \mathbf{1} \le \mathbf{X}_2 \le \frac{|D|^T\mathbf{X}}{2} \text{ and } \mathbf{X}_2 \text{ is binary} \quad (31)$$

	Global Optimal		Greedy Algorithm	
Number	PMU	Performance	PMU	Performance
$c = 4$	2,8,10,13	103060	2,4,10,13	784.42
$c = 5$	4,5,6,7,9	402336	2,4,8,10,13	226960
$c = 6$	4,5,6,7,8,9	1496600	2,4,7,8,10,13	827150
	Integer Programming Method		GDFSM	
Number	PMU	Performance	PMU	Performance
$c = 4$	2,6,7,9	72830	1,4,6,10	1.6407
$c = 5$	4,5,6,7,9	402336	1,4,6,10,14	1218.62
$c = 6$	4,5,6,7,8,9	1496600	1,4,6,8,10,14	349182

TABLE I
NUMERICAL RESULTS OF STANDARD IEEE 14 BUS SYSTEM

	Global Optimal		Greedy Algorithm	
Number	PMU	Performance	PMU	Performance
$c = 3$	2,6,9	25103	4,10,13	116.29
$c = 4$	4,5,6,9	112680	2,4,10,13	78442
$c = 5$	4,5,6,7,9	402336	2,3,4,10,13	286509
	Integer Programming Method		GDFSM	
Number	PMU	Performance	PMU	Performance
$c = 3$	2,6,9	25103	1,4,6	0.1575
$c = 4$	4,5,6,9	112680	1,4,6,10	164.07
$c = 5$	4,5,6,7,9	402336	1,4,6,10,14	121862

TABLE II
NUMERICAL RESULTS OF MODIFIED 14 BUS SYSTEM

VI. NUMERICAL RESULTS

In this section, we present results based on implementing the greedy algorithm and integer programming optimization methods to determine the optimal PMU deployment for several different test systems. We first consider the standard IEEE 14 bus [13] and modified 14 bus system for a transmission grid, which removes the bus 8 from the standard IEEE 14 bus system. After that, we consider the Feeder 4 bus and Feeder 13 bus system [14], which represent test systems in a distribution network. We assume that the admittance of all the edges in each test system is all unity. According to [12], the integer programming optimization equations formulated above can be solved by YALMIP toolbox embedded in MATLAB.

The results for each test system is shown in Table I to Table IV respectively. The PMU deployment and state estimation performance achieved from greedy algorithm and integer programming optimization method are compared with the global optimal solution obtained by exhaustive searching method, which would not be suitable for a large power grid due of the huge computation complexity. We also compare the results of our algorithm with those achieved from GDFSM [5] that only focuses on searching among PMU deployments that ensure full observability, to show that our methods are able to achieve better state estimation performance.

From the results, in the test system of meshed transmission grid, we see the greedy algorithm cannot guarantee to achieve the optimal solution with global maximal $\Omega(\Psi)$. However, its convergence speed is fast enough. On the other hand, though the integer programming optimization method may not lead to global optimal solution either, its state estimation performance is still better than that from greedy algorithm and GDFSM. Furthermore, in contrast with exhaustive search, integer

	Global Optimal		Greedy Algorithm	
Number	PMU	Performance	PMU	Performance
$c = 1$	2	0.01	2	0.01
$c = 2$	2,3	5	2,3	5
$c = 3$	1,2,3	21	1,2,3	21
	Integer Programming Method		GDFSM	
Number	PMU	Performance	PMU	Performance
$c = 1$	2	0.01	2	0.01
$c = 2$	2,3	5	2,4	4
$c = 3$	1,2,3	21	1,2,4	18

TABLE III
NUMERICAL RESULTS OF FEEDER 4 BUS SYSTEM

	Global Optimal		Greedy Algorithm	
Number	PMU	Perf.	PMU	Perf.
$c = 6$	3,4,5,8,9,10	1197	3,4,5,8,9,10	1197
$c = 7$	3,4,5,8,9,10,12	4701	3,4,5,8,9,10,12	4701
$c = 8$	3,4,5,6,8,9,10,12	18445	3,4,5,6,8,9,10,12	18445
	Integer Programming Method		GDFSM	
Number	PMU	Perf.	PMU	Perf.
$c = 6$	3,4,5,8,9,10	1197	2,4,6,8,10,13	408
$c = 7$	3,4,5,6,8,9,10	4701	2,4,6,8,9,10,13	3483
$c = 8$	3,4,5,6,8,9,10,11	18445	2,3,4,6,8,9,10,13	15336

TABLE IV
NUMERICAL RESULTS OF FEEDER 13 BUS SYSTEM

programming is able to find the optimal PMU placement for large grids. In the test system of a radial distribution network, our methods lead to global optimal PMU deployment while GDFSM does not.

In summary, selection of an optimization approach depends on the acceptable tradeoff between convergence speed and state estimation performance. If we require more accurate state estimation, we may choose integer programming optimization method, especially in the transmission grid. On the other hand, if we put more focus on faster convergence speed, the greedy algorithm is a viable option.

VII. CONCLUSION

In this paper, we considered how to find the unique optimal PMU deployment with the best state estimation performance. We formulated the state estimation model, which is able to describe the states of all the buses in the network regardless of whether the full observability is achieved or not. Then we proposed greedy algorithm and integer programming optimization method to solve this problem. The performance of these two methods was presented in the numerical results.

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