# Effects of Random Rough Surface on Absorption by Conductors at Microwave Frequencies

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***Abstract*—The effects of a random rough surface on the absorption by a metallic surface at microwave frequencies are analyzed by using two methods: the analytic small perturbation method and the numerical method of moments method. The results show significant difference between absorption of a rough surface and that of a smooth surface. The absorption depends on the root mean square height, correlation length, and correlation function of the random rough surface. The similarities with and differences from Morgan's classical result and the Hammerstad and Bekkadal formula are discussed. It is shown that for multiscale rough surfaces, saturation of absorption does not occur, or occurs at much higher frequencies.**

***Index Terms*—Conductors, method of moments (MoM), perturbation methods, rough surfaces.**

## I. Introduction

THE roughness of the interfaces between layers, especially in microelectronic packaging based on organic materials, is often used to facilitate the adherence of the copper structures to the dielectrics. Since the speed of interconnects is rapidly increasing to the multi-GHz region, the roughness of the surface can have significant effects on signal integrity. Existing commercial software tools do not allow users to model the surface roughness of the substrates accurately. Presently, the common results are due to Morgan's classical paper [1] and the Hammerstad and Bekkadal formula [2]. There are recent analyses [3] the results of which are consistent with Morgan's. However, in Morgan's analysis and in these other analyses, a periodic rough surface is used, often with rectangular grooves. The Hammerstad and Bekkadal formula is

$$\frac{P_{a,\mathrm{rough}}}{P_{a,\mathrm{smooth}}} = 1 + \frac{2}{\pi}\arctan\left[1.4\left(\frac{h}{\delta}\right)^2\right]. \qquad (1)$$

The absorption ratio in (1) depends only on the root mean square (RMS) height $h$ of the rough surface profile, besides the skin depth $\delta$.

In this letter, we use the model of random rough surfaces. A random rough surface is characterized by RMS height, correlation length, and correlation function. The advantage of using a random rough surface model is the resemblance with the surfaces occurring in copper interconnects. Furthermore, by measuring surface profiles, the random rough surface characteristics can be extracted quantitatively. The absorption due to these profiles is calculated by two methods: the analytic small perturbation method of Rayleigh and Rice to second order (SPM2, where 2 stands for second order) [4], [5] and the numerical method of moments (MoM). The result of absorption based on SPM2 is in terms of the spectral density of the random rough surface. The use of SPM2 is needed because absorption deals with power and SPM2 conserves energy to second order.

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## II. Random Rough Surfaces

For a two-dimensional (2-D) problem of random rough surface, the height function $f(x)$ is treated as a stationary random process. The two point ensemble average of the random process is

$$\langle f(x_1)f(x_2)\rangle = h^2 C\left(|x_1 - x_2|\right) \qquad (2)$$

where $h^2C(x)$ is the correlation function. Two common correlation functions are the Gaussian correlation function with $C(x) = \exp(-x^2/\mathrm{l}^2)$ and exponential correlation function with $C(x) = \exp(-x/\mathrm{l})$, where l is the correlation length. The exponential correlation profile appears significantly rougher than that for the Gaussian correlation function. In generating the roughness profiles [6], we use the spectral density function $W(k_x)$ which is the Fourier transform of the correlation function. The spectral density of the Gaussian correlation function is given by $W(k_x) = (h^2\mathrm{l}/2\sqrt{\pi})\exp(-k_x^2\mathrm{l}^2/4)$ and that of the exponential correlation function by $W(k_x) = (h^2\mathrm{l}/\pi)(1 + k_x^2\mathrm{l}^2)^{-1}$ [4]. Note that because the spectral density of the exponential correlation function decays slowly with increasing $k_x$, the surface contains multiscale roughness.

## III. Analytic Small Perturbation Method

Consider a 2-D problem with a random rough surface profile $z = f(x)$. Let $\psi$ be the magnetic field that is in the $y$ direction. Then

$$\psi(x,z) = \int_{-\infty}^{\infty} dk_x \exp(-jk_x x + jk_{1z}z)\widetilde{\psi}(k_x) \qquad (3)$$

where $k_{1z} = \sqrt{k_1^2 - k_x^2}$ and $k_1 = (1-j)/\delta$. Here, $\delta$ is the skin depth $\delta = \sqrt{2/\omega\mu\sigma}$, and $\sigma$ is the conductivity of the conductor, $\mu$ is its magnetic permeability, and $\omega = 2\pi f$ is the angular frequency. We use a second order small perturbation method, setting

$$\widetilde{\psi}(k_x) = \widetilde{\psi}^{(0)}(k_x) + \widetilde{\psi}^{(1)}(k_x) + \widetilde{\psi}^{(2)}(k_x). \qquad (4)$$

Following Morgan [1], we assume that the magnetic field on the surface $z = f(x)$ is a constant $H_0$

$$H_0 = \int_{-\infty}^{\infty} dk_x \exp\left[-jk_x x + jk_{1z}f(x)\right]\widetilde{\psi}(k_x). \qquad (5)$$

Balancing (5) to second order gives

$$\widetilde{\psi}^{(0)}(k_x) = H_0\delta(k_x) \quad (6.1)$$

$$\widetilde{\psi}^{(1)}(k_x) = -jk_1H_0F(k_x) \quad (6.2)$$

$$\widetilde{\psi}^{(2)}(k_x) = H_0\int_{-\infty}^{\infty} dk_x' F(k_x - k_x')F(k_x') \times\left(-k_1k_{1z}' + \frac{k_1^2}{2}\right) \quad (6.3)$$

where $F(k_x)$ is the Fourier transform of $f(x)$ and $\delta(k_x)$ is the Dirac delta function.

The power absorbed by the conductor, for a given width $w$ in the $y$ direction and length $L$ in the $x$ direction, is obtained from

$$P_a = \frac{w}{2\sigma}\mathrm{Re}\int ds\frac{\partial\psi}{\partial n}\psi^*. \quad (7)$$

Calculating the power absorbed to second order leads to

$$\begin{aligned}P_a = \frac{w}{2\sigma}\mathrm{Re}\Bigg\{&\int dx\left(-\frac{df}{dx}\right)\int_{-\infty}^{\infty} dk_x \exp(-jk_xx)\\ &\times[1+jk_{1z}f(x)]\left[\widetilde{\psi}^{(0)}(k_x)+\widetilde{\psi}^{(1)}(k_x)\right](-jk_x)\\ &\times\int_{-\infty}^{\infty} dk_x' \exp(jk_x'x)\left[1-jk_{1z}'^*f(x)\right]\\ &\times\left[\widetilde{\psi}^{(0)*}(k_x')+\widetilde{\psi}^{(1)*}(k_x')\right]\Bigg\}+\frac{w}{2\sigma}\\ \times\mathrm{Re}\Bigg\{&\int dx\int_{-\infty}^{\infty} dk_x\exp(-jk_xx)\\ &\times\left[1+jk_{1z}f(x)-\frac{k_{1z}^2f^2(x)}{2}\right]\\ &\times\left[\widetilde{\psi}^{(0)}(k_x)+\widetilde{\psi}^{(1)}(k_x)+\widetilde{\psi}^{(2)}(k_x)\right](jk_{1z})\\ &\times\int_{-\infty}^{\infty} dk_x'\exp(jk_x'x)\left[1-jk_{1z}'^*f(x)-\frac{k_{1z}'^{*2}f^2(x)}{2}\right]\\ &\times\left[\widetilde{\psi}^{(0)*}(k_x')+\widetilde{\psi}^{(1)*}(k_x')+\widetilde{\psi}^{(2)*}(k_x')\right]\Bigg\}. \quad (8)\end{aligned}$$

Taking the ensemble average of (8), using the property

$$\langle F(k_{1x})F^*(k_{2x})\rangle = \delta(k_{1x}-k_{2x})W(k_{1x}) \quad (9)$$

and simplifying the results gives

$$\langle P_a\rangle = \frac{wL}{2\sigma\delta}|H_0|^2 \times\left\{1+\frac{2h^2}{\delta^2}\left[1-\frac{\delta}{h^2}\int_{-\infty}^{\infty} dk_xW(k_x)\mathrm{Re}\sqrt{k_1^2-k_x^2}\right]\right\}. \quad (10)$$

The first term in (10) is the absorption for a smooth surface, $\langle P_a\rangle_{\text{smooth}}$; the ratio of rough to smooth surface absorption is

$$\frac{\langle P_a\rangle}{\langle P_a\rangle_{\text{smooth}}} = 1+2\left[\frac{h^2}{\delta^2}-\frac{2}{\delta}\int_0^{\infty}dk_xW(k_x)\mathrm{Re}\sqrt{-\frac{2j}{\delta^2}-k_x^2}\right]. \quad (11)$$

A SPM2 method was also used by Sanderson [7] and the result for internal surface displacement is expressed in terms of spectral density. However, (11) has an integrand that asymptotically approaches $W(k_x)/(\delta^2k_x)$ as $k_x$ becomes large. Thus the integral is convergent even for the exponential correlation function.

## IV. Numerical Approach Using MoM

Next, we solve the well known surface integral equation [6]

$$\frac{1}{2}\psi(\bar{r}')+\int_S ds\psi(\bar{r})\hat{n}\cdot\nabla g_1(\bar{r},\bar{r}')-\int_S dsg_1(\bar{r},\bar{r}')\hat{n}\cdot\nabla\psi(\bar{r}) = 0 \quad (12)$$

where $\psi = H_0$ on the surface $z = f(x)$ and

$$g_1(\bar{r}',\bar{r}'') = -\frac{j}{4}H_0^{(2)}(k_1|\bar{r}'-\bar{r}''|). \quad (13)$$

The first integral in (12) is taken as the principal value with an infinitesimally small piece subtracted out from the domain of integration.

We apply the periodic boundary condition with the period $L$. It is a valid approximation to random rough surface scattering provided that the period contains many peaks and valleys and many correlation lengths, i.e., $L \gg \mathrm{l}$ [6]. Using MoM, the matrix equation is

$$\sum_n A_{mn}^{(1)}u_{1n} = -H_0\sum_n B_{mn}^{(1)}. \quad (14)$$

In the above equation, the matrix elements are as follows. For $m \neq n$,

$$A_{mn}^{(1)} = K_{10}(\bar{r}_m,\bar{r}_n)\Delta x+\sum_{\substack{q=-\infty\\q\neq0}}^{\infty}K_{10}(\bar{r}_m,\bar{r}_n+qL\hat{x})\Delta x \quad (15)$$

$$B_{mn}^{(1)} = -K_{1N0}(\bar{r}_m,\bar{r}_n)\Delta x + \sum_{\substack{q=-\infty\\q\neq0}}^{\infty}\Delta x\left[-K_{1N0}(\bar{r}_m,\bar{r}_n+qL\hat{x})\right] \quad (16)$$

where $\bar{r}_m$ is the center of the $m$th patch in the MoM discretization and $\gamma$ is Euler's constant.

For $m = n$

$$A_{mm}^{(1)} = -\frac{j}{4}\Delta x\left\{1-\frac{j2}{\pi}\left[\ln\left(\frac{\gamma}{4}k_1\Delta x\right)-1\right]\right\} + \sum_{\substack{q=-\infty\\q\neq0}}^{\infty}K_{10}(\bar{r}_m,\bar{r}_m+qL\hat{x})\Delta x \quad (17)$$

$$B_{mm}^{(1)} = -\frac{1}{2}+\sum_{\substack{q=-\infty\\q\neq0}}^{\infty}\Delta x\left[-K_{1N0}(\bar{r}_m,\bar{r}_m+qL\hat{x})\right] \quad (18)$$

where

$$K_{10}(\bar{r}',\bar{r}'') = g_1(\bar{r}'',\bar{r}') \quad (19)$$

$$K_{1N0}(\bar{r}',\bar{r}'') = \hat{n}''\cdot\nabla''g_1(\bar{r}'',\bar{r}'). \quad (20)$$

Note that the matrix elements include an infinite number of summations because of the use of periodic Green's functions. The unknowns to solve correspond to the normal derivative of magnetic field on the surface. After the surface integral equation is solved, the ratio of power absorption is calculated by

$$\frac{P_{a,\text{rough}}}{P_{a,\text{smooth}}} = \frac{\delta}{L|H_0|^2}\mathrm{Re}\int ds\frac{\partial\psi_1}{\partial n}\psi_1^* \quad (21)$$

where the integration is over the length $L$ in the $x$ direction. It can be shown that as $\delta\to\infty$, the ratio in (21) approaches unity. In the numerical implementation, we take $L = 10\mathrm{l}$ and surface discretization is chosen as $\Delta x = \min\{h/10, \mathrm{l}/10, \delta/10\}$.

To calculate the average power absorption, we use a Monte–Carlo simulation approach. We generate a large number of realizations of rough profiles. Solving the MoM equation we then calculate the absorption ratio for every realization and the average absorption is computed. In the simulations conducted in this letter 600 realizations are used.

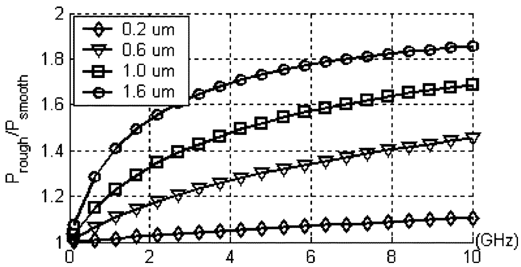


Fig. 1. Power absorption ratio as a function of frequency from SPM2: Gaussian correlation function ($l = h$) with varying RMS height.

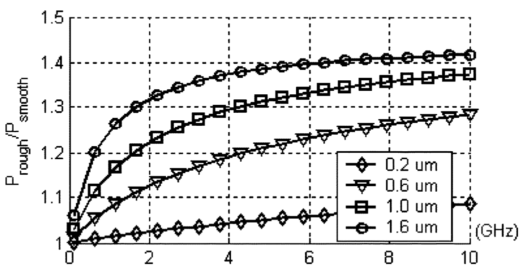


Fig. 2. Power absorption ratio from SPM2: Gaussian surface ($l$ = 1.5 h).

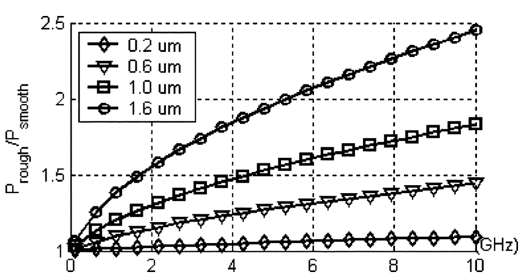


Fig. 3. Power absorption ratio from SPM2: Exponential surface ($l$ = 1.5 h).

## V. Results and Discussion

### A. SPM2 Results

Figs. 1–3 illustrate the SPM2 results of power absorption ratio. In Fig. 1, the results are for a Gaussian correlation function with $l = h$. The RMS height $h$ varies from 0.2 to 1.6 $\mu$m. We note that the absorption ratio increases with frequency. It also increases with RMS height. In Fig. 2, the results are repeated for the case of $l = 1.5$ h. The absorption ratios are smaller than those of Fig. 1 because a larger correlation length gives a smoother surface. The results show that for the Gaussian correlation function, saturation is consistent with Morgan's findings. In Fig. 3, the results are illustrated for surfaces with exponential correlation functions exhibiting larger absorption than surfaces with Gaussian correlation function. Also, the absorption ratio results do not saturate which is distinctly different from the results given by Morgan and the Hammerstad and Bekkadal formula. The results of Figs. 1–3 show that the absorption depends on all three of the roughness characteristics: RMS height, correlation length, and correlation function.

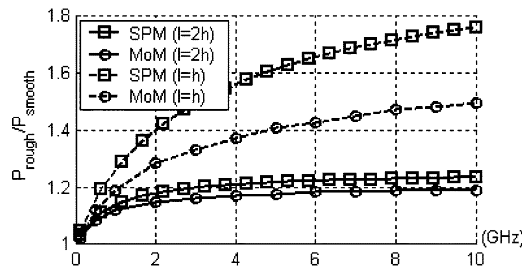


Fig. 4. SPM2 versus MoM: Gaussian surface ($h$ = 1.2 $\mu$m).

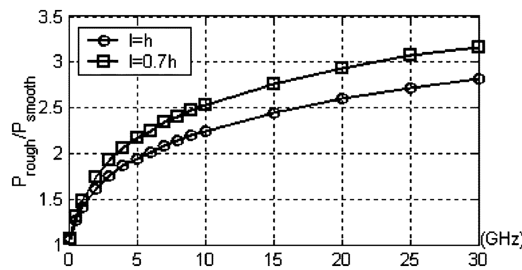


Fig. 5. Power absorption ratio from MoM: Exponential surface ($h$ = 2 $\mu$m).

### B. SPM2 and MoM Comparison

As seen in Section III, the small perturbation method only requires computation of one integral and thus it is convenient for evaluating power absorption for different random rough surfaces. However, SPM2 involves a second-order approximation and its accuracy needs to be verified. Fig. 4 compares SPM2 results with MoM results. The modeled surface profiles are Gaussian with $h$ = 1.2 $\mu$m and $l = h$ ($l$ = 2 h). In case of a smoother surface ($l$ = 2 h), the SPM2 and MoM results are in good agreement. The results also show saturation close to 1.2 for the absorption ratio. In the case of a rougher surface ($l = h$), the absorption ratio for SPM2 is larger than for MoM. The difference is about 17% at 10 GHz.

Fig. 5 shows the simulated power absorption using MoM. The modeled surface profile parameters are $h$ = 2 $\mu$m and $l$ = 0.7 h ($l = h$) with an exponential correlation function. In this case, the roughness of the conductor is quite significant so that the power absorption ratio goes up to around 3 at 30 GHz. In contrast, the classical Hammerstad and Bekkadal formula would approach an asymptotic value of only 2.

Extraction of spectral density from measured data and correlation between measurement and theoretical model are currently being investigated. Three–dimensional (3-D) surface data can be obtained with an optical scanning system with a 90-nm resolution. We are also extending the approach to 3-D problems.

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