# Maximizing Influence in Competitive Environments: A Game-Theoretic Approach

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**Abstract.** Ideas, ranging from product preferences to political views, spread through social interactions. These interactions may determine how ideas are adopted within a market and which, if any, become dominant. In this paper, we introduce a model for Dynamic Influence in Competitive Environments (DICE). We show that existing models of influence propagation, including linear threshold and independent cascade models, can be derived as special cases of DICE. Using DICE, we explore two scenarios of competing ideas, including the case where a newcomer competes with a leader with an already-established idea, as well as the case where multiple competing ideas are introduced simultaneously. We formulate the former as a Stackelberg game and the latter as a simultaneous-move game of complete information. Moreover, we show that, in both cases, the payoff functions for both players are submodular, leading to efficient algorithms for each player to approximate his optimal strategy. We illustrate our approach using the Wiki-vote social network dataset.

**Keywords:** Social network, influence propagation, noncooperative game

## 1 Introduction

Ideas spread rapidly through human social interactions, especially when enabled by modern technology, including blogs, online social networking sites, and mobile and pervasive computing. Such interactions can be used to convey information to the public at little direct cost. In politics, both traditional social networks (such as groups of politically like-minded people) and new, online social networks (such as Facebook and Twitter) have been instrumental in spreading revolutionary sentiment [12]. Commercial marketing campaigns have also leveraged social media, with companies using online social networks to enhance word-of-mouth effects in advertising [13].

In the applications listed above, multiple, competing ideas, potentially interfering with one another, may propagate simultaneously through the network. This competition may take different forms [9]. In the *leader-follower* (or *Stackelberg*) case, a well-established idea, such as a market-leading product or prevailing political belief, is challenged by newcomers, denoted as *followers*. Alternatively, two or more ideas may be introduced simultaneously, as in a political election with two or more parties.

In all of these competitive scenarios, the success or failure of each idea may depend on how the idea is introduced into the network, in particular who the initial holders of the idea (denoted as *seeds*) are. For instance, an idea espoused by the owner of a popular blog may reach a large number of people, while an idea held by a handful of isolated individuals may not spread at all [11]. In addition, there may be scenarios where no idea is able to completely dominate the other ideas. Effective introduction of an idea into a social network therefore requires an understanding of how competing ideas will propagate through the network, as well as a tractable framework for choosing the set of nodes in the network that must initially hold the idea. At present, however, while there are formulations of the competitive influence maximization problem [2, 7], they do not lead to computationally tractable solution algorithms for the three classes of players listed above.

Our contributions in this paper are two-fold. As our first contribution, we introduce a model for Dynamic Influence in Competitive Environments (DICE). Under DICE, each individual adopts an idea based on observations of his neighbors' current beliefs, leading to a Markov model of idea propagation. Unlike existing approaches, DICE allows nodes to switch between adopted ideas over time. This allows modeling of the case where a new idea is able to overtake or replace an existing, well-established idea. We further show how to leverage the Markovian properties of our proposed model to compute the expected number of individuals holding each idea, as well as the probability that a certain individual holds a given idea, in steady-state.

As our second contribution, we develop game-theoretic formulations for competition between two ideas within the Stackelberg and simultaneous competitive environments described above. Our influence model leads to an average-case optimization problem for each player. We show that, for the case of a social network with strongly connected components (as in [1]), the objective function for each player is *submodular*. As a result, solution algorithms can be developed for each player that approximate the optimal strategy up to a provable bound.

The rest of this paper is organized as follows. In Section 2, we review the related work on influence propagation. In Section 3, we define and analyze our proposed influence propagation models. In Sections 4 and 5, we introduce non-cooperative game formulations for the Stackelberg and simultaneous competition models, respectively. Simulations are presented in Section 6, and Section 7 concludes the paper and gives directions for future work.

## 2 Related Work and Preliminaries

### 2.1 Influence propagation models

The first mathematical model of influence propagation was the threshold model of [5]. This model assumes that an individual will adopt a given idea if a threshold number of its acquaintances adopt the idea first. The threshold model can be motivated both by sociological observation and as the equilibrium of a non-cooperative game between the individuals comprising the network. Another class of propagation models is based on *cascading* phenomena, in which each individual attempts to convince his or her neighbor of the idea, succeeds with a certain probability, and otherwise fails and does not try again [4].

Markov models for propagation of belief through a network have been proposed in the context of gossip and rumor spreading [3, 1]. In such models, each individual's belief is represented by a real number. The individuals reach consensus on a global belief by taking randomized weighted averages of their neighbors' beliefs. While our approach uses a Markov model of influence propagation, we study the case where ideas are competing and mutually exclusive, thus ruling out averaging and consensus.

### 2.2 Maximizing spread of influence

The problem of influence maximization in social networks was first proposed in [11], in the context of marketing. In [6], the authors analyzed the problem of choosing an optimal set of $k$ seed nodes in order to maximize the spread of influence. It was shown that, for a generalized influence model taking both the cascade and threshold models as special cases, the influence maximization problem is *submodular*, enabling the use of greedy approximation heuristics.

Extensions to the case of multiple competing ideas have been explored recently. In [2], the cascade model is extended to competing ideas, and it is shown that, for a follower, the influence maximization problem is submodular. Strategies for leaders, however, are only computable under specific assumptions about the network topology. In [7], the connection between influence maximization and competitve facility location was explored, with the observation that, under a basic diffusion model related to [4], approximating the optimal strategy for the leader is NP-hard.

### 2.3 Background on Submodular Functions

The notion of *submodularity* will be used to derive solution algorithms for the formulations of Section 4. The notion of submodularity is defined as follows.

**Definition 1.** *Let $V$ be a finite set. A function $f : 2^V \rightarrow \mathbf{R}$ is* submodular *if, for any $S \subseteq T \subseteq V$ and any $v \in V \setminus T$,*

$$f(S + v) - f(S) \geq f(T + v) - f(T) \tag{1}$$

Definition 1 can be understood as a "diminishing returns" property, in which the incremental utility of adding an element to a set decreases as the set grows. For additional background on submodular functions, including the following lemma, see [10].

**Lemma 1.** *A nonnegative weighted sum of submodular functions is submodular.*

## 3 Proposed Influence Propagation Model: DICE

In this section, the Dynamic Influence in Competitive Environments (DICE) model is defined. The social network is defined by a graph $G = (V, E)$, where $V$ is the set of $n$ individuals (also referred to as *nodes*) and $E$ is the set of social relationships, with $(i, j) \in E$ if individual $i$ has influence over individual $j$. $N(i)$ is the set of individuals who have influence over $i$.

A set of $m$ ideas, indexed $\mathcal{I} = \{I_1, \ldots, I_m\}$, is present in the network. $I_k$ is assumed to have an originator $\mathcal{O}_k$. At each time $t$, each individual $i \in V$ has a state $x_i(t) \in \{0, \ldots, m\}$, where $x_i(t) = k$ if $i$ holds idea $I_k$ at time $t$ and $x_i(t) = 0$ if $i$ has not adopted an idea at time $t$. Further, it is assumed that each node may be aware of multiple ideas, even if it only holds one of them. Let $\mathcal{I}_i(t) \subseteq \mathcal{I}$ be the set of ideas that $i$ is aware of at time $t$.

The propagation of ideas under DICE proceeds as follows. At time $t = 0$, let $V_k$ be the set of individuals with $x_i(0) = k$. All nodes $i \in V \setminus \cup_{k=1}^{m} V_k$ have $x_i(0) = 0$. At each subsequent time step $t$, each individual $i$ chooses a node $j \in N(i) \cup \{i\}$ with probability $d_{ij} > 0$. If $j = i$, then $i$ chooses an idea $I_l \in \mathcal{I}_i(t)$ with probability $P_i(k, l)$, where $k = x_i(t)$. Individual $i$ then updates its state to $x_i(t+1) = l$. If $j \neq i$, then $i$ sets $x_i(t+1) = x_j(t)$ with probability $P_{ij}(x_i(t), x_j(t))$, and sets $x_i(t+1) = x_i(t)$ otherwise. In either case, $i$ updates $\mathcal{I}_i$ according to $\mathcal{I}_i(t+1) = \mathcal{I}_i(t) \cup \{x_j(t)\}$.

This approach can be generalized to include probabilistic social network models, such as those in [6, 4], as follows. Suppose that there is a base topology $G = (V, E_0)$, and let $\mathcal{P}$ be a probability distribution, where $\mathcal{P}(E)$ is the probability that a given edge set $E \subseteq E_0$ occurs. Then for a given realization $E \subseteq E_0$, let

$$d_{ij}^{E} = \frac{d_{ij}}{\sum_{(i,j) \in E} d_{ij}} \tag{2}$$

while the values of $P_i(k, l)$ and $P_{ij}(x_i(t), x_j(t))$ remain unchanged.

DICE contains several existing influence models as special cases. These connections are summarized in Table 1.

| **Existing model** | **Parameters** |
|---|---|
| Triggering model [6] | Number of ideas $m = 1$ |
| Generalized linear threshold model [5] | $P_i(k, l) \equiv \frac{1}{\lvert \mathcal{I}_i(t) \rvert}$, choice of $P_{ij}(k, l)$ |
| Independent cascade model [2] | $\mathcal{P}(E) = \prod_{(u,v) \in E} p_{U,V}$ |

**Table 1.** Existing influence models as realizations of DICE. The triggering model is equivalent in steady-state, while the remaining models have the same dynamics.

### 3.1 Distribution of ideas in steady-state

The eventual popularity of a given idea can be studied by examining the asymptotic distribution of ideas. This steady-state distribution determines the probability $\pi_i(k)$ that individual $i$ holds idea $I_k$ for large values of $t$, given an initial distribution $V_1, \ldots, V_m$. The total expected number of nodes holding ideas $I_1, \ldots, I_m$ can then be used to evaluate the effectiveness of using $V_1, \ldots, V_m$ as seed nodes. The following theorem gives necessary conditions for this distribution to exist.

**Theorem 1.** *Suppose that, for each $i \in V$ and $k, l \in \mathcal{I}_i(t)$, $d_{ii} > 0$ and $P_i(k,l) > 0$. Then for a given collection of seed nodes $V_1, \ldots, V_m$, the proposed influence propagation model converges to a unique stationary distribution for $(x_1, \ldots, x_n)$, where $\pi_i(k|V_1, \ldots, V_m)$ denotes the stationary probability of node $i$ holding idea $I_k$. Furthermore, let*

$$\mathcal{I}_i(V_1, \ldots, V_m) \triangleq \{I_k \in \mathcal{I} : \exists \text{ path from } V_k \text{ to } i \ \} \tag{3}$$

*If $G$ can be decomposed into strongly connected components, then*

$$\pi_i(k|V_1, \ldots, V_m) = \pi_i(k|V_1', \ldots, V_m') \tag{4}$$

*for any $V_1, \ldots, V_m$ and $V_1', \ldots, V_m'$ satisfying $\mathcal{I}_i(V_1, \ldots, V_m) = \mathcal{I}_i(V_1', \ldots, V_m')$.*

Due to space constraints, a full proof is not given here. A sketch of the proof is as follows. By the definition of DICE, every individual becomes aware of the ideas held by its neighbors, even if it does not immediately adopt the ideas. This, together with the assumptions that $d_{ii} > 0$ and $P_i(k,l) > 0$, implies that $i$ becomes aware of every idea in $\mathcal{I}_i$ within finite time. Hence for sufficiently large $t$, the vector $(x_1(t), \ldots, x_n(t))$ can take any value in the space $\mathcal{S} = \mathcal{I}_1 \times \cdots \times \mathcal{I}_n$. Moreover, since $d_{ii} > 0$ and $P_i(k,l) > 0$, there is a nonzero probability of transitioning into any state $s \in \mathcal{S}$ at each time step. This implies that the chain, restricted to $\mathcal{S}$, is irreducible and aperiodic, and hence has a unique stationary distribution over $\mathcal{S}$. Finally, if $G$ has strongly connected components, then for every component $G_l$ and $i, j \in G_l$, the relationship $\mathcal{I}_i = \mathcal{I}_i(t) = \mathcal{I}_j(t) = \mathcal{I}_j$ holds for sufficiently large values of $t$, regardless of which specific nodes are in the sets $V_k \cap G_l$.

## 4 Problem Formulation: Leader-Follower Model

In this section, we formulate the competition between leader and follower ideas as a Stackelberg game. Analysis and algorithms for both the leader and follower strategies are provided as well. Although DICE can be applied to an arbitrary finite number of ideas, the case of two ideas is considered in the following sections in order to ensure simplicity of the solution algorithms.

### 4.1 Game Definition

The leader-follower game is defined as follows.

**Definition 2.** *The* Stackelberg competing ideas game *consists of two players $\mathcal{O}_k$, $k = 1, 2$, each of which owns a competing idea $I_k$. One of the players (without loss of generality, assume it is $\mathcal{O}_1$) selects a set of individuals $V_1$ to implant with $I_1$ at time 0. The second player, $\mathcal{O}_2$, observes $V_1$ and then chooses a set of individuals $V_2 \subseteq V \setminus V_1$ in which to implant $I_2$. Player $\mathcal{O}_k$'s payoff $U_k$ is given by*

$$U_k(V_1, V_2) = \sum_{E \subseteq E_0} \left[\mathcal{P}(E) \left(\sum_{i \in V} \pi_i(k|V_1, V_2, E)\right)\right] - c|V_k| \tag{5}$$

*where $c > 0$ is the cost associated with implanting an idea and $\pi_i(k|V_1, V_2, E)$ is the steady-state probability that individual $i$ will hold idea $I_k$ given initial sets $V_1$ and $V_2$ and edge set $E$.*

Under this formulation, the goal of player $\mathcal{O}_2$ is to find the set $V_2^*(V_1)$ satisfying

$$V_2^*(V_1) = \arg \max_{V_2 : V_2 \cap V_1 = \emptyset} U_2(V_1, V_2) \tag{6}$$

Meanwhile, the goal of player $\mathcal{O}_1$ is to find the set $V_1^*$ satisfying

$$V_1^* = \arg \max_{V_1} U_1(V_1, V_2^*(V_1)) \tag{7}$$

### 4.2 Solving Stackelberg Game for Follower

The goal of the follower is to find the set of seed nodes that maximizes the number of individuals holding $I_2$ in steady-state, given knowledge of the leader's seed nodes $V_1$. The follower's optimal strategy when $G$ is deterministic is given by Proposition 1. When the topology is probabilistic, the follower's strategy can be found by using submodular optimization techniques, as described in Theorem 2.

**Proposition 1.** *Suppose that the underlying interaction topology $G$ can be divided into strongly connected components $G_1, \ldots, G_L$. Then the follower's best response consists of a single node $v_l$ from each connected component $G_l$ satisfying*

$$\sum_{i \in V} \pi_i(2|V_1, v_l) > c \tag{8}$$

*Proof.* First, note that, for each component $G_l$ and each $i \in G_l$, $\pi_i(2|V_1, V_2)$ depends only on whether $G_l \cap V_2$ is nonempty by Theorem 1. This, coupled with the fact that

$$\sum_i \pi_2(2|V_1, V_2) - c|G_l \cap V_2| \leq \sum_i \pi_2(2|V_1, V_2) - c \tag{9}$$

implies that any $V_2$ with $|G_l \cap V_2| > 1$ is suboptimal.

Now, $U_2(V_1, V_2|E)$ can be rewritten as

$$U_2(V_1, V_2|E) = \sum_{l=1}^{L} \left(\sum_{i \in G_l} (\pi_i(2|V_1, V_2)) - c|G_l \cap V_2|\right) \tag{10}$$

Since propagation of ideas in disconnected components is independent, each term of the outer sum of (10) can be considered independently. Thus $|G_l \cap V_2| = 1$ is optimal iff the corresponding term of (10) is positive, which occurs iff (8) holds. □

**Theorem 2.** *When the interaction topology is stochastic and* $V_1$ *is fixed,* $U_2(V_1, V_2)$ *is a submodular function of* $V_2$*.*

*Proof.* By Proposition 1, the incremental gain from adding $v \in G_l$ to $V_2$ is equal to

$$U_2(V_1, V_2 + v) - U_2(V_1, V_2) = \begin{cases} \sum_{i \in G_l} \pi_i(2|V_1, v) - c, & V_2 \cap G_l = \emptyset \\ -c, & \text{else} \end{cases} \tag{11}$$

Now, if $S \subseteq T$ and $T \cap G_l = \emptyset$, then $S \cap G_l = \emptyset$. Hence

$$U_2(V_1, S + v) - U_2(V_1, S) \geq U_2(V_1, T + v) - U_2(V_1, T) \tag{12}$$

proving that $U_2(V_1, \cdot)$ is submodular. In general, $U_2$ is given by

$$U_2(V_1, V_2) = \sum_{E \subseteq E_0} U_2(V_1, V_2|E)\mathcal{P}(E) \tag{13}$$

which is a nonnegative weighted sum of submodular functions, and hence is submodular. □

The submodularity of $U_2(V_1, \cdot)$ implies that (6) is a *submodular maximization problem*. Although the submodular maximization problem is NP-hard, algorithms have been proposed that are guaranteed to achieve a provable approximation bound in polynomial time [10].

An algorithm for solving (6) is as follows. Initialize $V_2^0 = \emptyset$. At each subsequent iteration $t$, find a node $v^*$ satisfying

$$v^* = \arg \max_{v \in V \setminus (V_1 \cup V_2^t)} U_2(V_1, V_2 + \{v\}) - U_2(V_1, V_2) \tag{14}$$

If $U_2(V_1, V_2^t \cup \{v^*\}) - U_2(V_1, V_2^t) > 0$, then set $V_2^{t+1} = V_2^t \cup \{v^*\}$, increment $t$, and continue. Otherwise the algorithm terminates. A pseudo-code description of the algorithm is given in Figure 1.

Submodularity of $U_2(V_1, \cdot)$ and Proposition 4.1 of [10], yields the following proposition on the optimality of the algorithm.

**Proposition 2.** *The algorithm of Figure 1 returns a set* $\tilde{V}_2$ *such that*

$$U_2(V_1, V_2^*) - U_2(V_1, \tilde{V}_2) \leq c|\tilde{V}_2| \tag{15}$$

```
FOLLOWER_SEED_NODE_SELECTION
Input: Set V_1, topology G = (V, E), distribution P(E)
Output: Set V_2
V_2^0 ← ∅
t ← 0
while(1)
    v ← arg max_{v∈V\V_1} U_2(V_1, V_2^t + v) − U_2(V_1, V_2^t)
    if U_2(V_1, V_2^t + v) > U_2(V_1, V_2^t)
        V_2^{t+1} ← V_2^t ∪ {v}
        t ← t + 1
    else
        break
return V_2^t
```

**Fig. 1.** Pseudo-code description for submodular maximization of follower payoff.

### 4.3 Solving Stackelberg Game for Leader

As a preliminary, the following lemma describes the leader's payoff for a given set of seed nodes $V_1$.

**Lemma 2.** *For fixed topology $G = (V, E)$, the payoff for the leader is given by*

$$U_1(V_1, V_2^*(V_1)|E) = \sum_{l:G_l \cap V_1 \neq \emptyset} \sum_{i \in G_l} w_i - |V_1|c \tag{16}$$

*where $w_i$ is given by*

$$w_i = \begin{cases} \pi_i(1|I_1, I_2), & \sum_{i \in G_l} \pi_i(2|I_1, I_2) > c \\ \pi_i(1|I_1) & \textit{else} \end{cases} \tag{17}$$

*and $\pi_i(1|I_1, I_2)$ and $\pi_i(1|I_1)$ are the stationary probability that $i$ holds idea $I_1$ when $\mathcal{I}_i = \{I_1, I_2\}$ and $\mathcal{I}_i = \{I_1\}$, respectively.*

*Proof.* By Proposition 1, the follower's best response is to add a node $v_l \in G_l$ iff $\sum_{i \in G_l} \pi_i(2|I_1, I_2) > c$. In this case, the leader's payoff is $\sum_{i \in G_l} \pi_i(1|I_1, I_2)$ by Definition 2. Otherwise, the nodes in $G_l$ only become aware of $I_1$, and so the leader's payoff is $\sum_{i \in G_l} \pi_i(1|I_1)$. □

This leads to the following theorem, analogous to Theorem 2.

**Theorem 3.** *$U_1(V_1, V_2^*(V_1))$ is submodular as a function of $V_1$.*

*Proof.* By Lemma 2, the incremental gain from adding $v$ to $V_1$ when $G$ is deterministic is given by

$$U_1(V_1 + v, V_2^*(V_1 + v)) - U_1(V_1, V_2^*(V_1)) = \begin{cases} \sum_{i \in G_l} w_i - c, & V_1 \cap G_l = \emptyset \\ -c, & \text{else} \end{cases} \tag{18}$$

Hence the incremental gain is positive iff $V_1 \cap G_l = \emptyset$. Given $S \subseteq T \subseteq V$, $T \cap G_l = \emptyset$ implies that $S \cap G_l = \emptyset$. Thus

$$U_1(S+v, V_2^*(S+v)) - U_1(S, V_2^*(S)) \geq U_1(T+v, V_2^*(T+v)) - U_1(T, V_2^*(T)) \quad (19)$$

as desired. As in Theorem 2, when $G$ is stochastic, $U1$ is a nonnegative weighted sum of submodular functions, and is therefore submodular. □

Theorem 3 implies that an algorithm analogous to that in Figure 1 can be used to solve the leader's optimization problem (7). A pseudo-code description of the algorithm is contained in Figure 2.

LEADER_SEED_NODE_SELECTION
**Input:** Topology $G = (V, E)$, distribution $\mathcal{P}(E)$
**Output:** Set $V_1$
$V_1^0 \leftarrow \emptyset$
$t \leftarrow 0$
**while**(1)
  $v \leftarrow \arg\max_{v \in V} U_1(V_1^t + v, V_2^*(V_1^t + v))$
  **if** $U_1(V_1^t + v, V_2^*(V_1^t + v)) > U_1(V_1^t, V_2^*(V_1^t))$
    $V_1^{t+1} \leftarrow V_1^t \cup \{v\}$
    $t \leftarrow t + 1$
  **else**
    **break**
**return** $V_1^t$

**Fig. 2.** Pseudo-code description for submodular maximization of leader payoff.

## 5 Problem Formulation: Simultaneous Model

Under the simultaneous-move game, the originators of competing ideas simultaneously choose sets of seed nodes. This models the case where two ideas are introduced at the same time, or, more generally, when neither player is able to observe the other's choice of seed nodes before introducing his idea.

**Definition 3.** *The simultaneous-move game consists of two players $\mathcal{O}_k$, $k = 1, 2$, each of which owns a competing idea $I_k$. The players simultaneously select sets $V_k$ of individuals to implant with idea $I_k$ at time $0$. (If the players choose the same individual, then that individual adopts one of the ideas but is aware of both of them). Player $\mathcal{O}_k$'s payoff is given by*

$$U_k(V_1, V_2) = \sum_{i \in V} \pi_i(k|V_1, V_2) - c|V_k| \quad (20)$$

*where $c$ and $\pi_i$ are defined as in Definition 2.*

In what follows, analysis of the simultaneous-move game under DICE is provided. The first observation is that, when the topology is deterministic, the game can be decomposed into a set of $L$ games, each played on a different connected component $G_l$ of the social network $G$. For a given component $G_l$, each player chooses whether or not to choose $V_k$ such that $G_l \cap V_k \neq \emptyset$. The resulting payoff matrix is given by Table 2, where $E_k = \sum_{i \in G_l} \pi_i(k)$, $H$ denotes the case where $G_l \cap V_k \neq \emptyset$, and $H'$ denotes the case where $G_l \cap V_k = \emptyset$.

| | $H$ | $H'$ |
|---|---|---|
| $H$ | $(E_1 - c, E_2 - c)$ | $(\lvert G_l\rvert - c, 0)$ |
| $H'$ | $(0, \lvert G_l\rvert - c)$ | $(0, 0)$ |

**Table 2.** Payoffs of $\mathcal{O}_1$ and $\mathcal{O}_2$ for simultaneous-move game.

The following theorem describes the Nash equilibria of the game.

**Theorem 4.** *For the simultaneous-move game with a single component $G_l$,*
(i) *If $|G_l| < c$, then the game has a unique Nash equilibrium of $(H', H')$.*
(ii) *If $E_1 > c$ (resp. $E_2$) and $E_2 < c$ (resp. $E_1$), then the game has a unique Nash equilibrium of $(H, H')$ (resp. $(H', H)$).*
(iii) *If $E_k > c$ for $k = 1, 2$, there are two pure strategy Nash equilibria and one Nash equilibrium in mixed strategies.*

*Proof.* Points (i) and (ii) follow by inspection of the payoff matrix, noting that the equilibria Pareto dominate the other possible strategies. When the conditions of (iii) hold, $(H, H')$ and $(H', H)$ are Nash equilibria by inspection. To find the mixed Nash equilibrium, note that it occurs when both parties are indifferent between playing $H$ and $H'$. Let $p_k$ denote the probability that player $k$ plays $H$. Then player 1's payoff from playing $H$ is $p_2(E_1 - c) + (1 - p_2)(|G_l| - c)$ while the payoff from playing $H'$ is 0. Setting these equal yields $p_2 = \frac{|G_l| - c}{|G_l| - E_1}$. Thus the mixed strategy equilibrium is given by

$$p_1 = \frac{|G_l| - c}{|G_l| - E_2}, \qquad p_2 = \frac{|G_l| - c}{|G_l| - E_1} \tag{21}$$

as desired. □

## 6 Simulation Study

In this section, a simulation study of the leader-follower game of Section 4 is presented. Simulations were performed using Matlab on the Wiki-vote dataset [8]. A link $(i, j)$ exists if user $i$ voted in favor of user $j$ becoming an administrator. The original data set had $|V| = 7115$; in order to reduce runtime, randomly chosen subsets of $V$ were used for simulation. It was assumed that each edge in $E_0$ was added to $E$ with probability chosen uniformly at random from $[0, 0.5]$.

For each edge $(i, j)$, the probability that individual $i$ changes from $I_1$ to $I_2$ (or vice versa) based on $j$'s input was chosen uniformly at random from $[0, 1]$. The probability of an individual $i$ spontaneously switching between ideas was chosen uniformly at random from $[0, 0.05]$. The remaining simulation parameters are summarized in Table 3.

| **Parameter** | **Values Used** |
|---|---|
| Number of nodes, $n$ | $n = 100, 200, 300, 400, 500, 600, 700$ |
| Number of ideas, $m$ | 2 |
| Probability of self-determination $d_{ii}$ | 0.5 |
| Probability of $i$ choosing $j$, $d_{ij}$ | $\frac{0.5}{\lvert N(i)\rvert}$ |
| Cost of adding an individual to $V_k$, $c$ | 5 (low cost) and 15 (high cost) |

**Table 3.** Simulation parameters

Figure 3 shows the number of individuals holding ideas $I_1$ and $I_2$ in steady-state for different values of $n$ and $c$. The payoffs of both players increase with network size. However, in most cases, the payoff of the leader exceeds the payoff of the follower. This is because the follower must choose whether to compete with the leader for influence over highly-connected clusters of individuals. When the follower has no incentive to do so, the leader may automatically gain control of these clusters at minimal cost.

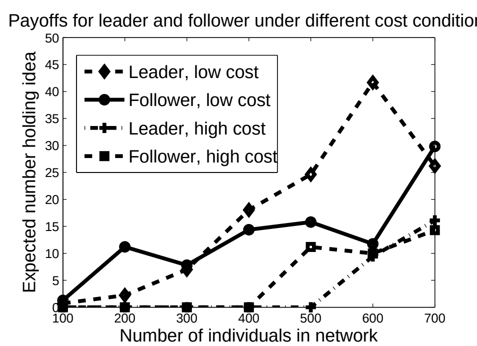


**Fig. 3.** Simulation of the leader-follower game using the Wiki-vote dataset. The payoff for each player increases with network size; the rate of increase depends on the cost $c$. The leader's payoff typically exceeds the follower's.

Increasing the cost $c$ gives each player less incentive to target individual nodes. Figure 3 suggests that there is a cutoff $n(c)$ on the network size in order for idea originators to be willing to introduce their ideas. This may be interpreted as a "barrier to entry" for ideas to enter the marketplace [9].

## 7 Conclusions and Future Work

In this paper, the problem of maximizing influence of competing ideas was studied. The Dynamic Influence in Competitive Environments (DICE) model, which uses Markov processes to model the propagation of ideas through a social network, was introduced. Based on DICE, game-theoretic models of competition between ideas were developed, including a Stackelberg game modeling the interaction between a leader and a follower in a marketplace, as well as a model for simultaneous introduction of ideas. It was shown that computationally tractable algorithms can be used to approximate the solution for both players.

In our proposed formulation it was assumed that both players have complete, full knowledge of the network topology and each other's attributes. Our plan of future work is to develop models of competition with incomplete information. We will also extend the static games analyzed in this paper to dynamic games, in which the owner of each idea adapts his strategy in response to the actions of his competitors. Another direction of future work is to improve on the speed and accuracy of the solution algorithms of Section 4.

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