

# Transactions Briefs

## On Upper Bounds of the Equivalent Oscillator and Notch-Filter Circuits: A Non-Commutative Group Theoretic Approach

P. Raadhakrishnan, Andrew E. Yagle, B. V. Rao, and  
John E. Dorband

**Abstract**—In this paper we show that the problem of synthesizing equivalent oscillators and notch-filters from the knowledge of a given parent circuit is identical to that of finding the isomorphisms of a non-commutative group. It is also shown that many of the results established by earlier methods can easily be explained using the theory developed in this paper. The results derived in this paper show that earlier results do not lead to the best possible upper bound. The corrected upper bound is derived by counting the members of the non-commutative group. A well-known family of circuits is used to illustrate the theory developed.

### I. INTRODUCTION

In the recent past there have been attempts to group RC oscillators and notch-filters so that additional equivalent circuits can be generated from the knowledge of a given parent circuit [7]–[14]. Among these efforts, [7]–[12] are applicable only to oscillator circuits. A unified synthesis framework for oscillators and notch-filters alike was presented in [14]. However, the results reported in [15] indicated that the upper bound is greater than the existing bounds.

In this paper a synthesis method based on non-commutative group theory that extends the earlier reported results on upper bounds is presented. Also demonstrated is the fact that the elements of the non-commutative structure are natural extension of the results reported earlier by other investigators. The results reported in this paper have the advantage of synthesizing a stable circuit from an unstable parent circuit.

Let  $S$  be a set of  $n$  elements  $x_1, x_2, x_3, \dots, x_n$ . A 1:1 mapping  $\Phi$  acting on  $S$  is represented as

$$\Phi: \begin{pmatrix} x_1 & x_2 & x_3 & \dots & x_n \\ x_{i1} & x_{i2} & x_{i3} & \dots & x_{in} \end{pmatrix} \quad (1)$$

where  $x_{ik}$  is the image of  $x_k$  under  $\Phi$  and every  $x_{ik} \in S$ . A mapping  $\Phi$  is called decomposable if it can be expressed as a composition of two or more maps. For example,  $\Phi = \Phi_2 \Phi_1$  means that the action of  $\Phi$  is equivalent to the action of  $\Phi_1$  followed by the action of  $\Phi_2$ .

### II. THEORETICAL BASIS OF THE SYNTHESIS PROBLEM

#### 2.1. Derivation of the Network Aspects

A popular generalized canonical RC oscillator (notch-filter) structure is shown in Fig. 1. The operational amplifier (OA) employed in this circuit is assumed to be ideal with infinite-gain

Manuscript received August 12, 1991; revised June 8, 1992. This work was supported by the Office of Naval Research under Grant N00014-90-J-1897. This paper was recommended by Associate Editor D. J. Allstot. The authors are with the Department of Electrical Engineering and Computer Science, University of Michigan, Ann Arbor, MI 48109. IEEE Log Number 9204227.

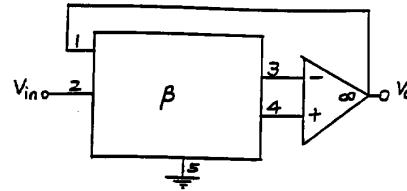


Fig. 1. Generalized canonical RC network under study.

mode, and  $\beta$  is assumed to be a passive RC network.  $V_{in}$  and  $V_o$  are the input and the output voltages of the circuit. Since OA is assumed to be ideal with  $\infty$  gain  $A$ ,  $\infty$  input impedance, and zero output impedance, from Fig. 1 it follows that  $A(V_3 - V_4) = V_o$ . Since  $A$  is infinite and  $V_o$  is finite,  $V_3 - V_4 = 0$ . Hence, the voltages at terminals three and four should be equal. As a consequence, any network connection between terminals three and four is redundant. The condition that the input impedance is  $\infty$  forces the currents  $I_3 = I_4 = 0$ . Let  $y_{ij}$  denote the admittance connected between the nodes  $i$  and  $j$  of the given passive network  $\beta$ .

1) *Oscillator Mode*: When the operating mode is that of self-sustained oscillations,  $V_{in} = V_o$ . With the assumptions made earlier, the following identity results [15]:

$$y_{31} * y_{44} - y_{41} * y_{33} = 0. \quad (2)$$

2) *Notch-Filter Mode*: For the notch-filter mode, at transmission null frequency  $\omega_o$ , the output voltage  $V_o$  is zero for a nonzero finite input voltage  $V_{in}$ . This condition, along with the additional observations made earlier, results in

$$y_{32} * y_{44} - y_{42} * y_{33} = 0. \quad (3)$$

Let  $\lambda$  be a set defined as  $\lambda: \{\theta_1, \theta_2, \gamma_1, \gamma_2\}$  where the elements of  $\lambda$  satisfy the constraint

$$\theta_1 * \theta_2 - \gamma_1 * \gamma_2 = 0. \quad (4)$$

Set  $\lambda$  can be used to denote the equations (2) or (3) due to their structural similarity. Depending on whether the circuit is an oscillator or a notch-filter,  $\lambda$  will take values from set  $\{y_{31}, y_{44}, y_{41}, y_{33}\}$  or set  $\{y_{32}, y_{44}, y_{42}, y_{33}\}$ , respectively. This observation leads to a unified approach to the synthesis problem as described in the next section.

### III. RELATIONSHIP BETWEEN GROUP ISOMORPHISMS OF A AND SYNTHESIS OF EQUIVALENT CIRCUITS

Consider set  $\lambda: \{\theta_1, \theta_2, \gamma_1, \gamma_2\}$  where the elements of  $\lambda$  satisfy constraint (4), which is called the reduced identity in the context of circuit theory [14]. Since this identity is common to all the

oscillators (notch-filters) that can be represented by the two-port structure in Fig. 1, any isomorphism of  $\lambda$  that leaves (4) unchanged will lead to an additional oscillator (notch-filter) circuit. Hence, set  $S$  of all isomorphisms of  $\lambda$  that leaves constraint (4) unchanged will enable one to find the upper bounds on the additional non-trivial equivalent circuits. In order to illustrate this point, let  $\Phi_i$  be an element of set  $S$ . Action of  $\Phi_i$  on  $\lambda$  will lead to  $\Phi_i(\lambda)$ :  $\{\Phi_i(\theta_1), \Phi_i(\theta_2), \Phi_i(\gamma_1), \Phi_i(\gamma_2)\}$ . Constraint (4) is changed to the form

$$\Phi_i(\theta_1) * \Phi_i(\theta_2) - \Phi_i(\gamma_1) * \Phi_i(\gamma_2) = 0. \quad (5)$$

Since  $\lambda$  has only four elements, the following are all the possible invariant isomorphisms:

$$\begin{aligned} \Phi_1: & \begin{pmatrix} \theta_1 & \theta_2 & \gamma_1 & \gamma_2 \\ \theta_1 & \theta_2 & \gamma_1 & \gamma_2 \end{pmatrix}; \Phi_2: \begin{pmatrix} \theta_1 & \theta_2 \\ \theta_2 & \theta_1 \end{pmatrix}; \Phi_3: \begin{pmatrix} \gamma_1 & \gamma_2 \\ \gamma_2 & \gamma_1 \end{pmatrix}; \\ \Phi_4: & \begin{pmatrix} \theta_1 & \theta_2 & \gamma_1 & \gamma_2 \\ \theta_2 & \theta_1 & \gamma_2 & \gamma_1 \end{pmatrix}; \\ \Phi_5: & \begin{pmatrix} \theta_1 & \theta_2 & \gamma_1 & \gamma_2 \\ \gamma_1 & \gamma_2 & \theta_1 & \theta_2 \end{pmatrix}; \\ \Phi_6: & \begin{pmatrix} \theta_1 & \theta_2 & \gamma_1 & \gamma_2 \\ \gamma_2 & \gamma_1 & \theta_2 & \theta_1 \end{pmatrix}; \Phi_7: \begin{pmatrix} \theta_1 & \theta_2 & \gamma_1 & \gamma_2 \\ \gamma_1 & \gamma_2 & \theta_2 & \theta_1 \end{pmatrix}; \\ \Phi_8: & \begin{pmatrix} \theta_1 & \theta_2 & \gamma_1 & \gamma_2 \\ \gamma_2 & \gamma_1 & \theta_1 & \theta_2 \end{pmatrix}. \end{aligned}$$

The following interesting properties are observed from the eight isomorphisms derived:

$$\Phi_1 = e = \text{identity mapping.}$$

$$\Phi_i * \Phi_i = \Phi_i^2 = e, i \in \{1, 2, 3, \dots, 8\}$$

$$\Phi_2 * \Phi_3 = \Phi_3 * \Phi_2 = \Phi_4; \quad \Phi_5 * \Phi_6 = \Phi_6 * \Phi_5 = \Phi_4; \quad \Phi_2 * \Phi_3 = \Phi_3 * \Phi_2 = \Phi_4;$$

$$\Phi_3 * \Phi_4 = \Phi_4 * \Phi_3 = \Phi_2; \quad \Phi_6 * \Phi_4 = \Phi_4 * \Phi_6 = \Phi_5; \quad \Phi_3 * \Phi_4 = \Phi_4 * \Phi_3 = \Phi_2;$$

$$\Phi_4 * \Phi_2 = \Phi_2 * \Phi_4 = \Phi_3; \quad \Phi_4 * \Phi_5 = \Phi_5 * \Phi_4 = \Phi_6; \quad \Phi_4 * \Phi_2 = \Phi_2 * \Phi_4 = \Phi_3.$$

It is easily verified that sets  $S_1, S_2,$  and  $S_3$  defined as  $S_1$ :  $\{\Phi_1, \Phi_2, \Phi_3, \Phi_4\}$ ,  $S_2$ :  $\{\Phi_1, \Phi_4, \Phi_5, \Phi_6\}$ , and  $S_3$ :  $\{\Phi_1, \Phi_4, \Phi_7, \Phi_8\}$  satisfy all the conditions of the Klein group presented in [14].

Another interesting property is that all the isomorphisms are self inverses. Hence, repeated application of any isomorphism on  $\lambda$  will be equivalent to applying the identity mapping or the original mapping itself, depending on whether the number of times the mapping is applied is odd or even, respectively. From a circuit theory viewpoint, this implies that the repeated application of a particular admittance transformation will not lead to more than one additional equivalent circuit. The same conclusion can be made by considering the two-element group, i.e., the identity element and any other given isomorphism from set  $S$ .

In order to construct the necessary non-commutative group, we have to note the following additional properties exhibited by the elements of set  $S$ :

$$\Phi_3 * \Phi_6 = \Phi_6 * \Phi_2 = \Phi_2 * \Phi_5 = \Phi_5 * \Phi_3 = \Phi_8$$

$$\Phi_3 * \Phi_5 = \Phi_5 * \Phi_2 = \Phi_2 * \Phi_6 = \Phi_6 * \Phi_3 = \Phi_7$$

$$\Phi_5 * \Phi_7 = \Phi_7 * \Phi_6 = \Phi_6 * \Phi_8 = \Phi_8 * \Phi_5 = \Phi_2$$

$$\Phi_7 * \Phi_5 = \Phi_6 * \Phi_7 = \Phi_8 * \Phi_6 = \Phi_5 * \Phi_8 = \Phi_3$$

$$\Phi_7 * \Phi_2 = \Phi_3 * \Phi_7 = \Phi_2 * \Phi_8 = \Phi_8 * \Phi_3 = \Phi_5$$

$$\Phi_2 * \Phi_7 = \Phi_7 * \Phi_3 = \Phi_8 * \Phi_2 = \Phi_3 * \Phi_8 = \Phi_6.$$

Since the elements of  $S$  are self inverses, any additional combination of the maps can be reduced to one of the above given forms. Hence,  $S$  is closed under the composition operation. A careful analysis shows that a composition of any two elements from different subsets  $S_1, S_2,$  and  $S_3$  is noncommutative. The composition operation plays the role of the binary operation for the group  $S$ . A number of nontrivial new solutions to condition (4) can now be computed using the group theoretic approach in more than one way. Given that the elements of  $S$  are the restricted isomorphisms of  $\lambda$ , the number of nontrivial additional solutions is equal to  $[\text{cardinality of } S - 1]$  (the  $[-1]$  being the removal of the identity element from  $S$ ). Hence, the number of additional new solutions is equal to seven.

The elements of  $\lambda$  take values from the two-port admittances that satisfy (2) for the oscillator mode (or (3) for the notch-filter). Therefore, computing all possible invariant isomorphisms of  $\lambda$  is equivalent to finding all the possible different two-port admittance transformations that leave the condition for oscillations unaffected. Hence, the number of additional equivalent circuits is equal to  $[\text{cardinality of } S - 1]$ .

#### IV. CIRCUIT THEORETIC INTERPRETATION OF THE PROPERTIES OF THE NON-COMMUTATIVE GROUP

1)  *$\Phi_1$  is an Identity Mapping:* After having chosen set  $\lambda$ :  $\{\theta_1, \theta_2, \gamma_1, \gamma_2\}$  from the sets  $\{y_{31}, y_{44}, y_{41}, y_{33}\}$  or  $\{y_{32}, y_{44}, y_{42}, y_{33}\}$ , applying  $\Phi_1$  to  $\lambda$  leads to the set  $\Phi_1(\lambda)$ :  $\{\Phi_1(\theta_1), \Phi_1(\theta_2), \Phi_1(\gamma_1), \Phi_1(\gamma_2)\}$ . However, since  $\Phi_1$  is the identity mapping ( $\lambda = \Phi_1(\lambda)$ ), condition (4) is unaffected by the action of  $\Phi_1$ . This means that when there is no rearrangement of the parent circuit, the original circuit will remain unaffected and there will be no new equivalent circuit.

2) *Every Element of Set  $S$  is a Self Inverse:* When  $i \neq 1$ ,  $\Phi_i$ , for  $i = 2, 3, \dots, 8$  will be a nontrivial isomorphism from  $(S, *)$ , where  $*$  is the composition operation. The action of  $\Phi_i$  on  $\lambda$  leads to reordering of the elements of  $\lambda$  and leave condition (4) fixed. Since  $\lambda$  contains the two-port admittances of the parent circuit, any reordering of  $\lambda$  leads to the reordering of the admittances and results in an additional equivalent circuit. This means that given a parent oscillator circuit, the action of a nontrivial  $\Phi_i$  readily synthesizes an additional new oscillator (notch-filter) circuit with the same operating frequency. However, if  $\Phi_i$  is applied to the new set  $\Phi_i(\lambda)$ , the resulting set is  $\Phi_i * (\Phi_i(\lambda))$  which can be rewritten as  $\Phi_i^2(\lambda) = \Phi_i(\lambda) = \lambda$ . Clearly the application of  $\Phi_i$  again will result in the set  $\Phi_i(\lambda)$ . This leads to the conclusion that a given nontrivial isomorphism leads to a unique equivalent circuit. Hence, the seven nontrivial isomorphisms lead to seven nontrivial equivalent circuits. This is not sufficient to conclude that the upper bound on the additional equivalent oscillator or notch-filter circuits is seven. The next observation leads to the sufficient part of the upper bound.

3) *Set  $S$  Forms a Non-Commutative Finite Group:* It was noted that the elements of set  $S$  form a non-commutative group. Set  $S$  can be written as the union of subsets  $S_1, S_2$  and  $S_3$  which form three different Klein groups. Hence, the total number of elements of the group  $S$  can be easily computed using the set

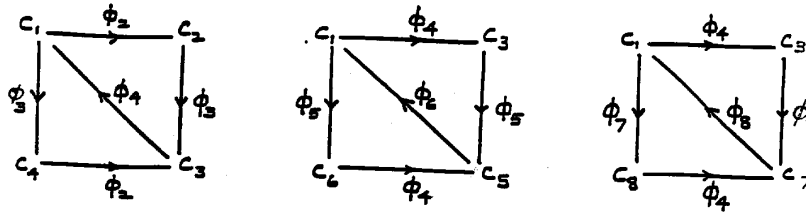


Fig. 2. Group isomorphic diagrams for oscillators.

theoretic concepts as

$$\begin{aligned} & |S_1| + |S_2| + |S_3| - |S_1 \cup S_2| - |S_2 \cup S_3| \\ & - |S_3 \cup S_1| + |S_1 \cap S_2 \cap S_3| \\ & = 4 + 4 + 4 - 2 - 2 - 2 + 1 = 7. \end{aligned}$$

This is much easier than going through an exhaustive search of all the possible combinations of the elements of  $S$ . It can be verified that there are 120 different ways in which these elements can be combined. Since some of them are non-commutative mappings, it is possible to have infinitely many different compositions which are equivalent to one of the eight elements of  $S$ . Hence, the number of nontrivial equivalent circuits is seven.

4) *Subset  $S_1$  is Complement to the Set  $S_2 \cup S_3$  in the Sense of Stability:* It can be shown that if the parent circuit is a stable circuit, subgroup  $S_1$  will generate all stable circuits and subgroups  $S_2$  and  $S_3$  will generate some unstable circuits. Since  $S$  has eight elements and the subgroup  $S_1$  has four elements, the number of stable circuits from a stable parent circuit will be four. This result holds for an oscillator and a notch-filter alike. A restricted version of this result for the case of a stable parent oscillator is available in [7]. Hence, even for a stable parent circuit, the results derived here are more general. However, the results gain additional strength from the proof that the present method can produce the identical number of stable circuits (notch-filters and oscillators) starting from an unstable parent circuit. This result is new and provides a tight upper bound on the stable circuits that can be synthesized from a stable or an unstable parent circuit. In the restricted case of oscillators, given an unstable parent circuit, results in [7] will produce only unstable circuits. Hence the result in [7] needs an additional condition that the parent circuit should be stable. Stability is not an issue in our procedure, as explained above. The next subsection shows how the non-commutative group results generalize the earlier reported results.

#### V. INTERPRETATION OF THE EARLIER RESULTS FROM NON-COMMUTATIVE FRAMEWORK

Most of the earlier results are applicable to either an oscillator or a notch-filter. Since the non-commutative result is applicable to both cases it is of interest to find out how well the established synthesis techniques relate to the present theory. Decomposition of set  $S$  into the commutative subgroups  $S_1$ ,  $S_2$ , and  $S_3$  becomes useful in this discussion.

In [7] it was shown that given a parent circuit employing an ideal OA, it is possible to generate three additional equivalent oscillators with the same operating frequency. The key assumption not explicitly mentioned in [7] is that the parent oscillator circuit needs to be stable. Hence, this method forces the inven-

tor to find at least one stable circuit by trial and error method. Given an unstable circuit, the method in [7] will generate only unstable circuits because it preserves the transfer function form. It can be shown that the method in [7] is identical to using subgroup  $S_1$  for a given stable parent circuit. Since the details will unduly extend the length of the paper we omit the proofs; they will be presented elsewhere.

Results in [8] can be obtained by choosing only  $S_1$  and  $S_2$  to construct additional equivalent circuits from the knowledge of a parent circuit. The upper bound on the additional equivalent oscillators was shown to be five in [8], and the nontrivial isomorphisms for sets  $S_1$  and  $S_2$  taken together were shown to be five in our earlier paper [14]. Since the results presented in [14] form a subset of the results in this paper, we conclude that taking any two subgroups from  $S_1$ ,  $S_2$ , and  $S_3$  leads to the upper bounds derived in [8]. The group isomorphic diagrams are presented in Fig. 2.

#### VI. ILLUSTRATIVE EXAMPLE

In this section we illustrate the theory developed in this paper and the claim that the non-commutative group method will enable one to synthesize the stable as well as unstable circuits for a given parent circuit. Since there are well-established results available for the case of oscillators we illustrate the theory for oscillators.

Assume that we are given the circuit in Fig. 3(a). In terms of the two-port admittances, the necessary constraint equation can be derived as

$$y_1 * y_3 - y_2 * y_4 = 0.$$

At this stage there are two choices for the admittances, as given below:

- 1) 
$$y_4 = \frac{SC_4}{SC_4R_4 + 1}, y_2 = \frac{1}{R_2}, y_3 = R_3 \text{ and } y_1 = \frac{1}{R_1} + SC_1$$
- 2) 
$$y_1 = \frac{SC_1}{SC_1R_1 + 1}, y_2 = \frac{1}{R_2}, y_3 = R_3 \text{ and } y_4 = \frac{1}{R_4} + SC_4.$$

Though both of these choices lead to the necessary conditions, only one choice leads to a stable circuit. In general, stability cannot be predetermined. Circuits have to be generated and then using simulation methods, the stable circuits are chosen. Let us consider the first choice of parameters. The corresponding circuit is shown in Fig. 3(a). Letting  $\theta_1 = y_1$ ,  $\theta_2 = y_3$ ,  $\gamma_1 = y_2$ , and  $\gamma_2 = y_4$ , we can construct the necessary non-commutative

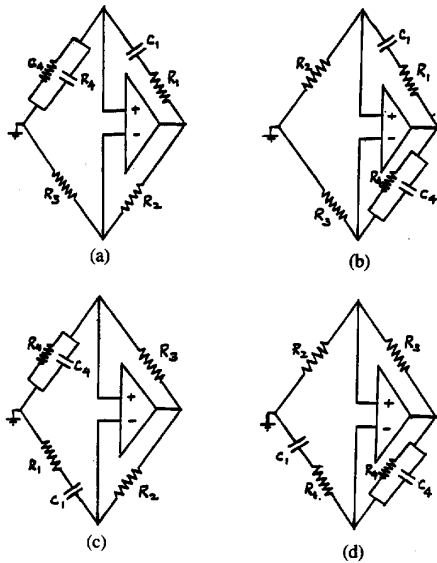


Fig. 3. Unstable Wien-Bridge oscillator family.

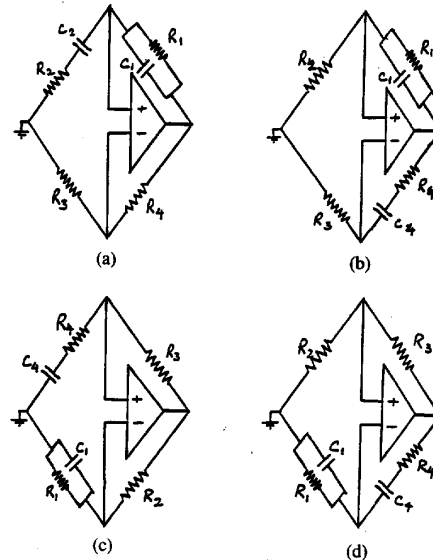


Fig. 4. Stable Wien-Bridge oscillator family.

group  $S$ . Elements of  $S$  are defined as follows:

$$\Phi_1: \begin{pmatrix} \theta_1 & \theta_2 & \gamma_1 & \gamma_2 \\ \theta_1 & \theta_2 & \gamma_1 & \gamma_2 \end{pmatrix}; \Phi_2: \begin{pmatrix} \theta_1 & \theta_2 \\ \theta_2 & \theta_2 \end{pmatrix}$$

$$\Phi_3: \begin{pmatrix} \gamma_1 & \gamma_2 \\ \gamma_2 & \gamma_1 \end{pmatrix}; \Phi_4: \begin{pmatrix} \theta_1 & \theta_2 & \gamma_1 & \gamma_2 \\ \theta_2 & \theta_1 & \gamma_2 & \gamma_1 \end{pmatrix}$$

$$\Phi_5: \begin{pmatrix} \theta_1 & \theta_2 & \gamma_1 & \gamma_2 \\ \gamma_1 & \gamma_2 & \theta_1 & \theta_2 \end{pmatrix}; \Phi_6: \begin{pmatrix} \theta_1 & \theta_2 & \gamma_1 & \gamma_2 \\ \gamma_2 & \gamma_1 & \theta_2 & \theta_1 \end{pmatrix}$$

$$\Phi_7: \begin{pmatrix} \theta_1 & \theta_2 & \gamma_1 & \gamma_2 \\ \gamma_1 & \gamma_2 & \theta_2 & \theta_1 \end{pmatrix}; \Phi_8: \begin{pmatrix} \theta_1 & \theta_2 & \gamma_1 & \gamma_2 \\ \gamma_2 & \gamma_1 & \theta_1 & \theta_2 \end{pmatrix}$$

Furthermore, sets  $S_1$ ,  $S_2$ , and  $S_3$  defined as  $S_1: \{\Phi_1, \Phi_2, \Phi_3, \Phi_4\}$ ,  $S_2: \{\Phi_1, \Phi_4, \Phi_5, \Phi_6\}$ , and  $S_3: \{\Phi_1, \Phi_4, \Phi_7, \Phi_8\}$  satisfy all the conditions of a Klein group [14].

Action of  $\Phi_2$  on the circuit shown in Fig. 3(a) leads to the circuit shown in Fig. 3(b). Action of  $\Phi_3$  on the circuits in Figs. 3(a) and (b) leads to the circuits shown in Figs. 3(c) and (d), respectively. It can be verified that the elements of subgroups  $S_1$  do not generate any additional circuits. We now show that the results in [7] will generate identical circuits. Applying *theorem 1* in [7] to the circuit in Fig. 3(a), we can generate the circuit in Fig. 3(b). Applying *theorem 2* in [7] to circuits in Figs. 3(a) and (b) leads to the circuits in Figs. 3(c) and (d), respectively. Since the method in [7] preserves the transfer function, it cannot generate any additional circuits.

However, the non-commutative group theoretic method can generate additional circuits using subgroups  $S_2$  and  $S_3$ . Action of  $\Phi_5$ ,  $\Phi_6$ ,  $\Phi_7$ , and  $\Phi_8$  on the circuit in Fig. 3(a) generates the circuits in Figs. 4(a), (b), (c), and (d), respectively. The circuits in Fig. 4 are all stable members of the well-known Wien Bridge oscillator family. We also note that applying subgroup  $S_1$  to the circuit in Fig. 4(a) leads to the remaining circuits in Fig. 4.

Hence, applying the results in [7] to the circuit in Fig. 4(a) will lead to the remaining circuits in Fig. 4.

A vast number of oscillator circuits are reported in [8]. These circuits can be generated and grouped using the theory presented here.

REFERENCES

- [1] S. K. Mitra, *Active Inductorless Filters*. New York: IEEE Press, 1971.
- [2] R. Schaumann, M. Soderstrand, and K. Laker, *Modern Active Filter Design*. New York: IEEE Press, 1981.
- [3] S. K. Mitra, *Analysis and Synthesis of Linear Active Networks*. New York: Wiley, 1969.
- [4] G. S. Moschytz, *Linear Integrated Networks*, vol. 2. New York: Van Nostrand Reinhold, 1974.
- [5] L. T. Brouton, *RC Active Circuits Theory and Design*. Englewood Cliffs, NJ: Prentice-Hall, 1980.
- [6] W. K. Chen, "Analysis of constraint active networks," *Proc. IEEE*, vol. 66, pp. 1655-1657, 1978.
- [7] N. Boutin, "On the identification of single amplifier single resistance controlled oscillators," *IEEE Trans. Circuits Syst.*, vol. CAS-31, pp. 1046-1048, Dec. 1984.
- [8] B. B. Bhattacharya, M. Sundara Murthy, and M. N. S. Swamy, "Systematic generation of canonical sinusoidal RC-active oscillators," *Proc. Inst. Elec. Eng.*, vol. 128, pp. 114-125, June 1981.
- [9] D. Hilberman, "Input and ground as components in active filters," *IEEE Trans. Circuit Theory*, vol. CT-20 pp. 540-547, Sept. 1973.
- [10] D. Hilberman, "An approach to the sensitivity and statistical variability of biquadratic filters," *IEEE Trans. Circuit Theory*, vol. CT-20, pp. 382-390.
- [11] Wierzba, "Op-amp relocation: A topological active network synthesis," *IEEE Trans. Circuits Syst.*, pp. 469-475, 1986.
- [12] T. S. Rathore, B. B. Bhattacharya, "Derivation of sinusoidal oscillators through network transformations," *IJETE*, vol-34, pp. 455-457, Dec. 1988.
- [13] P. Raadhakrishnan, *Systematic Realization of a Pair of Imaginary Axis Transmission Zeros, Internal Seminar Report*, Bombay: Indian Institute of Technology, 1987.
- [14] P. Raadhakrishnan and B. V. Rao, "Upper bound on the number of equivalent oscillator-notch filter circuits: A group theoretic approach," *Int. J. Circuit Theory Appl.*, vol. 18, pp. 521-533, 1990.
- [15] V. V. Prem Pyara, S. C. Dutta Roy, and S. S. Jamuar, "Identification and design of single amplifier single resistance controlled oscillators," *IEEE Trans. Circuits Syst.*, vol. CAS-30, pp. 176-181, 1983.