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Two-Degree-of-Freedom Damping Control of Driveline Oscillations Caused by Pedal Tip-In Maneuver

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Abstract— **Driveline oscillations caused by pedal tip-in maneuvers are uncomfortable for the passenger and stressing for the mechanical parts of the powertrain. In a drive-by-wire vehicle, the engine torque can be used as a control input signal to reduce driveline oscillations. Approaches based on feedback control alone are often insufficient to provide tracking performance in the presence of measurement delays. We design a feedforward controller using differential flatness that complements the feedback controller to improve tracking performance. For generality, our proposed feedforward control design is for an n-th order powertrain model. We show that the phase difference between the last two inertias is a flat output when the powertrain is undamped. Using this flat output, we design a feedforward control law to prevent oscillation during pedal tip-in. Simulation results show the improved tracking performance of our proposed approach in comparison to a feedback controller with a static prefilter.**

I. Introduction

Trade offs in powertrain control have become more challenging in recent years, due to increased demand for both high performance and fuel-efficiency [1]. To be more fuel-efficient, vehicle powertrains are designed as underdamped systems. A significant problem of underdamped powertrains is that rapid driving maneuvers induce torsion oscillations in the driveline. A particularly stressing maneuver for the powertrain is pedal tip-in. In a pedal tip-in maneuver, the driver requests a rapid acceleration change by quickly depressing the accelerator pedal. This acceleration change leads to torsion oscillation in the driveline. The passengers experience an undesired longitudinal jerking and mechanical parts of the powertrain are more stressed due to oscillations.

The progressive electrification of cars, in particular by drive-by-wire systems and automated manual transmissions, provides opportunities to improve drive quality. In a drive-by-wire vehicle, the engine torque is regulated by the electronic throttle control (ETC) system and hence can be used as a control input. The control signal provided by the engine torque must be designed to complete the pedal tip-in maneuver with few oscillations, and, if possible, without slowing down the drive dynamic.

Investigations of feedback control technique to dampen driveline oscillation have been widely studied [2], [3], [4], [5]. A feedback controller using time delayed output measurements, however, may destabilize the powertrain [6]. Measurement delays are induced by low sampling rate and communication delays between electronic control units. Feedback control laws that are robust to time delays can lead to degradation of tracking performance.

One approach to provide tracking performance in the presence of measurement delay is to design a feedforward controller that complements the feedback controller. Current work on joint design of feedforward and feedback controllers to damp driveline oscillations focuses on simplified, reduced-order models of the powertrain [7], [8], [9]. In [7], [8] a feedforward controller based on an approximated system dynamics is proposed, but detailed system dynamics are not considered. In [9] a flatness-based feedforward controller to damp driveline oscillation is presented. The approach of [9], however, is based on a reduced-order linear two-mass model and does not consider feedback control design.

A general approach based on a higher-order model of the powertrain dynamics would lead to improved drive quality for a broader class of powertrains, for instance, a two-order model can be suitable for control design of conventional powertrains, but for hybrid systems with several electric motors a higher-order model is required. In addition, a more accurate system model would translate to improved tracking performance of the feedforward controller. Currently, however, no such general approach to feedforward controller design for powertrains exists.

In this paper we present a two-degree-of-freedom control design approach, with focus on feedforward control design, for a general n^{th}-order powertrain model. Our approach is based on differential flatness theory, which provides an analytical approach to consider the whole system dynamics in feedforward control design. We make the following specific contributions:

- We derive a general state-space representation for a powertrain with f inertias that describes the dynamics of the phase and frequency difference between neighboring inertias of the powertrain.
- We show that in the undamped case, the phase difference between the last two inertias of the powertrain is a flat output by using the chain structure of the stiffness matrix of the derived system representation.
- We provide a two-degree-of-freedom control design. We formulate a feedforward control law, which inverts the undamped powertrain model exactly using differential flatness theory. For model mismatches and constant external disturbance, a linear quadratic integral (LQI) regulator is added to the control loop.
- We evaluate our approach through a simulation study.

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We compare our approach to an LQI feedback controller with a prefilter, and show that the flatness-based feedforward controller improves the tracking performance of the system.

The paper is organized as follows. Section II provides the system model for vehicle powertrains. In Section III a two-degree-of-freedom control loop with flatness-based feedforward controller and linear quadratic integrator feedback controller is presented. Section IV contains our simulation results and Section V concludes the paper.

II. Dynamic Model of a Vehicle Powertrain and Preliminaries

In this section we derive the equations of motion for a general powertrain model. The system model is used for the two-degree-of-freedom control design for vehicle powertrain in Section III. Background on differential flatness theory is also given.

A. Dynamic Model of a Vehicle Powertrain

Vehicle powertrains consist of various elements, including an engine (combustion and/or electric), clutch, gearbox, shafts, and wheels. These inertias are connected by elastic shafts and can therefore oscillate in different modes. A common technique to model a powertrain is to represent it as a multibody system [1], [10]. The inertias of the powertrain are assumed as lumped and coupled in a chain by spring and damper force elements. In our model, gear ratios will not be explicitly considered, since a powertrain model with gear ratios can be simplified by incorporating the gear ratios into the moments of inertia, spring functions, and damper functions [1], see Remark 1. The coupling functions of the spring and damper elements can be described as linear functions [10]. Due to the technology of drive-by-wire systems the engine torque can be used as the system input. We consider load torque on the wheels as a constant system disturbance.

Fig. 1 shows a detailed rear-driven powertrain model with seven lumped inertias [1]. The depicted inertias are: engine J_{eng}, clutch J_{cl}, transmission shaft J_{ts}, propeller shaft J_{ps}, final drive J_{fd}, drive shaft J_{ds}, and wheels J_w. The gear ratios are described by ig_1 and ig_2, and φ denotes the rotation phase of an inertia. The engine torque T_{eng} acts on the first inertia and a constant disturbance T_{load} acts on the last inertia.

For generality, we consider a powertrain model with f inertias. In the following, we derive the equation of motion for a system with f inertias and linear coupling functions by Newton's second law. Each inertia element has a constant moment of inertia $J_i \in \mathbb{R}$, time-varying phase $\varphi_i(t) \in \mathbb{R}$ and a time-varying frequency $\omega_i(t) \in \mathbb{R}$.

The dynamics of an inertia i for $i = 2, 3, \ldots, f-1$ with left neighbor $(i-1)$, right neighbor $(i+1)$ and constant coupling coefficients $c_i \in \mathbb{R}$, $d_i \in \mathbb{R}$ can be described by:

$$J_i\dot{\omega}_i + \sum_{j\in\{i-1,i+1\}} \left(d_i(\omega_i - \omega_j) + c_i(\varphi_i - \varphi_j)\right) = 0.$$

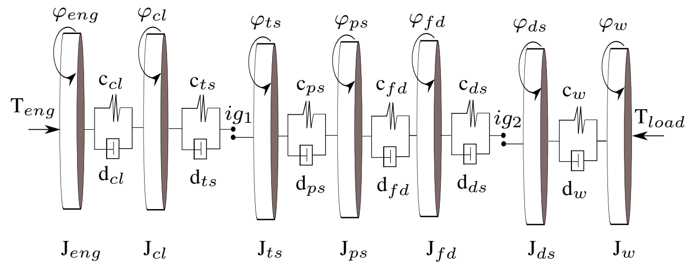

Fig. 1: Detailed powertrain model with lumped inertias: engine (eng), clutch (cl), transmission shaft (ts), propeller shaft (ps), final drive (fd), drive shaft (ds) and wheels (w); and gear ratios ig_1 and ig_2.

The dynamics of the first inertia (engine) with the engine torque as input u is

$$J_1\dot{\omega}_1 + d_1(\omega_1 - \omega_2) + c_1(\varphi_1 - \varphi_2) - u = 0.$$

The dynamics of the last inertia (wheels) with constant load torque δ is

$$J_f\dot{\omega}_f - d_{f-1}(\omega_{f-1} - \omega_f) - c_{f-1}(\varphi_{f-1} - \varphi_f) + \delta = 0.$$

Since we consider a chain of inertias, the equations of motion can be reduced by rewriting them as functions of the state:

$$x = \left[\Delta\varphi_1\ \Delta\varphi_2\ \ldots\ \Delta\varphi_{f-1}\ \Delta\omega_1\ \Delta\omega_2\ \Delta\omega_{f-1}\right]^T.$$

The system order becomes $n = 2f - 2$ and the linear SISO system dynamics is given by

$$(\Sigma): \quad \dot{x} = Ax + Bu + E\delta, \quad x(0) = x_0 \in \mathbb{R}^n \tag{1}$$

with system matrix

$$A = \begin{pmatrix} 0 & I_{f-1} \\ K & D \end{pmatrix},$$

where I is the $(f-1) \times (f-1)$ identity matrix.

In the undamped case the system matrix is

$$A_0 = \begin{pmatrix} 0 & I_{f-1} \\ K & 0 \end{pmatrix}. \tag{2}$$

The constant tridiagonal stiffness matrix K is given by

$$K = \begin{pmatrix} -c_1\alpha_1 & \frac{c_2}{J_2} & 0 & 0 & \cdots & 0 \\ \frac{c_1}{J_2} & -c_2\alpha_2 & \frac{c_3}{J_3} & 0 & \ddots & \vdots \\ 0 & \frac{c_2}{J_3} & -c_3\alpha_3 & \frac{c_4}{J_4} & \ddots & 0 \\ 0 & \ddots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \frac{c_{f-3}}{J_{f-2}} & -c_{f-2}\alpha_{f-2} & \frac{c_{f-1}}{J_{f-1}} \\ 0 & \cdots & 0 & 0 & \frac{c_{f-2}}{J_{f-1}} & -c_{f-1}\alpha_{f-1} \end{pmatrix}, \tag{3}$$

with $\alpha_k = \frac{1}{J_k} + \frac{1}{J_{k+1}}$ and $k = 1, 2, \ldots, (f-1)$. The damping matrix D has the same structure as K, but it has damper constants d_k instead of spring constants c_k. Damper elements in the damping matrix are small in comparison to spring elements in the stiffness matrix $d_k \ll c_k$ for $k = 1, 2, \ldots, f-1$, since powertrains are underdamped systems.

The input vector B and disturbance vector E are given as:

$$B = \begin{bmatrix} 0 & \ldots & 0 & \frac{1}{J_1} & 0 & \ldots & 0 \end{bmatrix}^T, \tag{4}$$

$$E = \begin{bmatrix} 0 & \ldots & 0 & 0 & 0 & \ldots & \frac{1}{J_f} \end{bmatrix}^T. \tag{5}$$

The input access is given via the f-th element in B and the disturbance influences the system via the n-th element in E.

Remark 1: Gear ratios can be incorporated into moment of inertia, load torque, damping and stiffness constant, as Fig. 2 illustrated. A gear ratio is defined as

$$ig = \frac{\omega_{in}}{\omega_{out}} = \frac{T_{out}}{T_{in}}.$$

An input angular velocity ω_{in} of the input gear is transmitted to the output angular velocity $\omega_{out} = \frac{\omega_{in}}{ig}$. On the other hand an input torque T_{in} is amplified to $T_{out} = T_{in} ig$. The equations of motion of a two-mass-model with gear ratio ig are:

$$\begin{aligned} J_1 \ddot{\varphi}_1 =& T_{eng} - \frac{1}{ig}\left[d(\frac{\dot{\varphi}_1}{ig} - \dot{\varphi}_2) + c(\frac{\varphi_1}{ig} - \varphi_2)\right], \\ J_2 \ddot{\varphi}_2 =& d(\frac{\dot{\varphi}_1}{ig} - \dot{\varphi}_2) + c(\frac{\varphi_1}{ig} - \varphi_2) - T_{load}. \end{aligned} \tag{6}$$

Substituting J_2, T_{load}, c, d, φ_2 by the new variables $J_2^{'}$, $T_{load}^{'}$, $c^{'}$, $d^{'}$, $\varphi_2^{'}$ with

$$J_2 = J_2^{'} ig^2, \quad T_{load} = T_{load}^{'} ig, \quad c = c^{'} ig^2, \quad d = d^{'} ig^2$$

and

$$\varphi_2 = \frac{\varphi_2^{'}}{ig}, \quad \dot{\varphi}_2 = \frac{\dot{\varphi}_2^{'}}{ig},$$

leads to:

$$\begin{aligned} J_1 \ddot{\varphi}_1 =& T_{eng} - \left[d^{'}(\dot{\varphi}_1 - \dot{\varphi}_2^{'}) + c^{'}(\varphi_1 - \varphi_2^{'})\right], \\ J_2^{'} \ddot{\varphi}_2^{'} =& d^{'}(\dot{\varphi}_1 - \dot{\varphi}_2^{'}) + c^{'}(\varphi_1 - \varphi_2^{'}) - T_{load}^{'}. \end{aligned} \tag{7}$$

Hence, without loss of generality, gear ratio is incorporated into moment of inertia $J_2^{'}$, load torque $T_{load}^{'}$, damping $d^{'}$ and stiffness constant $c^{'}$.

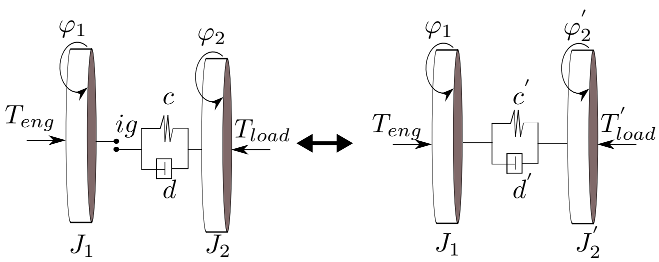

Fig. 2: Incorporating gear ratio into the moment of inertia $J_2^{'}$, load torque $T_{load}^{'}$, damping $d^{'}$ and stiffness constant $c^{'}$.

B. Differential Flatness

In the following, we give preliminaries from [11], [12], [13], [14] about differential flatness. We will use differential flatness techniques to derive a feedforward controller in Section III.

Differentially flat systems have the property that system state, input, and output can be fully described by the *flat output* and its derivatives. The system can then be represented with the new system state

$$x^* = \begin{bmatrix} y & \dot{y} & \ddot{y} & \ldots & y^{(n-1)} \end{bmatrix}^T. \tag{8}$$

The system representation with state (8) is in normal form and has no zero dynamics and the whole system dynamics can be determined by the new state x^*.

The definition of a general nonlinear differentially flat system is as follows:

Definition 1 ([11]): The nonlinear SISO system

$$\begin{aligned} \dot{x} =& f(x, u), \quad x(0) = x_0 \\ y =& h(x) \end{aligned} \tag{9}$$

with $x \in \mathbb{R}^n$ and $u, y \in \mathbb{R}$ is said to be *differentially flat*, if and only if there exists a flat output $z \in \mathbb{R}$, such that

- the flat output z is a function of the state variables x: $z = \lambda(x)$,
- the system state, input and output can be parametrized with z and its derivatives:

$$\begin{aligned} x =& \Psi_x\left(z, \dot{z}, \ldots, z^{(n-1)}\right), \quad u = \Psi_u\left(z, \dot{z}, \ldots, z^{(n)}\right), \\ y =& \Psi_y\left(z, \dot{z}, \ldots, z^{(n-r)}\right), \end{aligned}$$

where r is the relative degree of (9).

A main challenge in nonlinear feedforward control design based on differentially flatness theory is to find a flat output. For linear controllable systems, the flat output can be constructed by the Kalman controllability matrix. The following lemma gives a necessary and sufficient condition that a linear SISO system is flat:

Lemma 1 ([14]): A linear time-invariant SISO system is a flat system if and only if there exists a flat output $z \in \mathbb{R}$, such that

- the flat output z is a linear combination of the state variables x: $z = \lambda^T x$,
- the relative degree of the system with flat output z is equal to the system order.

III. Two-Degree-of-Freedom Control for a Vehicle Powertrain

In powertrain applications, the control loop has to be robust to output measurement delays and external disturbances and ensure good tracking performance. A two-degree-of-freedom control structure provides an additional degree of freedom to share these tasks. Both parts, feedforward and feedback, can be designed independently [15]. Our approach based on the two-degree-of-freedom control loop as shown in Fig. 3.

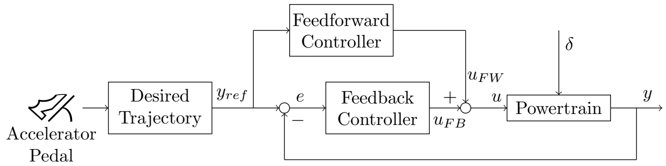

Fig. 3: Two-degree-of-freedom control structure with feedforward controller, feedback controller and powertrain plant.

After receiving a driver's requested signal from the accelerator pedal, a desired trajectory for set-point transition is generated or retrieved from a look-up table. We will show for an undamped powertrain model without disturbances that the feedforward controller guarantees that the system output y follows the generated desired trajectory y_{ref} exactly. Since the feedforward and feedback control designs are independent, any feedback controller that is suitable for the control problem can be chosen to handle model mismatches and disturbances. Our choice is a linear quadratic regulator with integrator (LQI). The integrator eliminates steady-state error in the presence of constant load disturbances.

In the following, we first present our main contribution: the feedforward control design for an undamped powertrain model. Since the feedforward controller needs a planned reference output trajectory, we discuss trajectory planning. We then describe the chosen feedback controller.

A. Feedforward Control Design for Undamped Powertrain Models

In this section, we first prove that the phase difference of the last two inertias is a flat output for the undamped case. We then present the feedforward control law using flatness theory.

1) Flat Output for an Undamped Vehicle Powertrain: Our feedforward control design is based on an undamped powertrain model without external disturbances:

$$(\Sigma_0): \quad \dot{x} = A_0 x + Bu, \quad x(0) = x_0 \in \mathbb{R}^n \tag{10}$$

with A_0 from (2), B from (4) and system order $n = 2f - 2$. External constant disturbances will be considered in the feedback controller design.

We neglect damping to design a feedforward controller, since damper constants are a difficult measurable dimension in automotive engineering and thus are not known exactly in general. Furthermore, damper constants are much smaller than spring constants and therefore the influence of the damping torque on the system dynamics is small. Additionally, in steady-state, damper elements have no effect on the system dynamics (Σ) from (1) or (Σ_0) from (10), as the frequency differences $\Delta\omega_i$ for $i = 1, 2, \dots, f-1$ are zero. System (Σ_0) can be seen as the nominal model of system (Σ). Feedforward control design based on a nominal model still improves output tracking performance, if the neglected system dynamics are sufficiently small [16].

From the necessary and sufficient conditions of Lemma 1 the flat output can be interpreted as an output which is 'farthest' away from the system input. As we consider a chain of inertias in our powertrain model, where the input is acting on the first inertia, it is intuitive that the state $\Delta\varphi_{f-1}$ (phase difference between the last two inertias) is a candidate to fulfill the relative degree conditions of Lemma 1.

In the following, we show that system (Σ_0) with system output $y = \Delta\varphi_{f-1}$ is flat. The output can be described by

$$y = \Delta\varphi_{f-1} = c^T x,$$

where c^T is a vector with a 1 in the $(f-1)$-th entry and zeros elsewhere.

Proposition 1: System (Σ_0) with output $y = \Delta\varphi_{f-1}$ is a flat system.

Proof: We use the necessary and sufficient conditions of Lemma 1 to proof the proposition. The n time derivatives of output y are calculated as follows:

$$\begin{aligned}
y &= \Delta\varphi_{f-1} = c^T x,\\
\dot{y} &= \Delta\omega_{f-1},\\
\ddot{y} &= \frac{1}{J_{f-1}} c_{f-2}\Delta\varphi_{f-2} - \alpha_{f-1}c_{f-1}\Delta\varphi_{f-1}\\
&= f\left(\Delta\varphi_{f-2}, \Delta\varphi_{f-1}\right),\\
\dddot{y} &= \frac{1}{J_{f-1}} c_{f-2}\Delta\dot{\varphi}_{f-2} - \alpha_{f-1}c_{f-1}\Delta\dot{\varphi}_{f-1}\\
&= \frac{1}{J_{f-1}} c_{f-2}\Delta\omega_{f-2} - \alpha_{f-1}c_{f-1}\Delta\omega_{f-1}\\
&= f\left(\Delta\omega_{f-2}, \Delta\omega_{f-1}\right),\\
y^{(4)} &= \frac{1}{J_{f-1}} c_{f-2}\Delta\dot{\omega}_{f-2} - \alpha_{f-1}c_{f-1}\Delta\dot{\omega}_{f-1}\\
&= f\left(\Delta\varphi_{f-3}, \Delta\varphi_{f-2}, \Delta\varphi_{f-1}\right),\\
y^{(5)} &= f\left(\Delta\omega_{f-3}, \Delta\omega_{f-2}, \Delta\omega_{f-1}\right),\\
y^{(6)} &= f\left(\Delta\varphi_{f-4}, \Delta\varphi_{f-3}, \Delta\varphi_{f-2}, \Delta\varphi_{f-1}\right),\\
&\vdots\\
y^{(n-1)} &= f\left(\Delta\omega_1, \Delta\omega_2, \dots, \Delta\omega_{f-1}\right),\\
y^{(n)} &= f\left(\Delta\dot{\omega}_1, \Delta\dot{\omega}_2, \dots, \Delta\dot{\omega}_{f-1}\right)\\
&= f\left(u, \Delta\varphi_1, \Delta\varphi_1, \dots, \Delta\varphi_{f-1}\right).
\end{aligned} \tag{11}$$

The n derivatives of y show a chain structure due to the tridiagonal stiffness matrix K. The input u appears first in the n^{th} derivative and it follows that the relative degree of the system is n. Therefore the necessary and sufficient conditions of Lemma 1 is fulfilled and system (1) with output $y = \Delta\varphi_{f-1}$ is a flat system. ∎

Since the system (1) is flat, Definition 1 implies that system state and system input can be parametrized with the flat output and its derivatives.

Proposition 2: The mapping from system state x to the new state x^* of the undamped system (Σ_0) from (10) is given by

$$x^* = \begin{pmatrix} y & \dot{y} & \ddot{y} & \dots & y^{(n-1)} \end{pmatrix}^T = Tx,$$

with transformation matrix

$$T = \begin{pmatrix} \kappa^T, & 0 \\ 0, & \kappa^T \\ K_{f-1}, & 0 \\ 0, & K_{f-1} \\ K_{f-1}^2, & 0 \\ 0, & K_{f-1}^2 \\ \vdots, & \vdots \\ K_{f-1}^{f-1} & 0 \\ 0, & K_{f-1}^{f-1} \end{pmatrix} \in \mathbb{R}^{n\times n}, \tag{12}$$

where K_{f-1}^k is the $(f-1)^{th}$-row of the k^{th} exponentiation of the stiffness matrix K and $\kappa^T = \begin{bmatrix} 0 & 0 & \dots & 0 & 1 \end{bmatrix} \in \mathbb{R}^{1\times(f-1)}$.

Proof: From the first $(n-1)$ equations of (11), the transformation matrix (12) of a flat system is given by

$$T = \begin{pmatrix} c^T \\ c^T A_0 \\ c^T A_0^2 \\ \vdots \\ c^T A_0^{n-1} \end{pmatrix}. \tag{13}$$

In the system (Σ_0) from (10), the power of A_0^i is for even $i \geq 2$

$$A_0^i = \begin{pmatrix} K^{\frac{i}{2}} & 0 \\ 0 & K^{\frac{i}{2}} \end{pmatrix} \tag{14}$$

and for odd $i \geq 3$

$$A_0^i = \begin{pmatrix} 0 & K^{\frac{i-1}{2}} \\ K^{\frac{i+1}{2}} & 0 \end{pmatrix}. \tag{15}$$

Due to the structure of the exponentiation of the system matrix A_0, it follows that (13) is fully described by the $(f-1)^{th}$ row of the k-th power of the stiffness matrix K with $k = 1, 2, \dots, (f-1)$ and the vector κ^T, which describes the dependence of the flat output on the phase differences. ∎

The inverse of the transformation matrix (12) leads to the parametrization of the system state

$$x = T^{-1}x^* = \Psi_x\left(y, \dot{y}, \dots, y^{(n-1)}\right).$$

The output is parametrized, since the output y itself is the flat output

$$\Psi_y(y) = y.$$

Remark 2: In powertrain applications, the output $\Delta\varphi_{f-1}$ might not be directly measurable. However, it is a suitable system output, since it can be calculated by the dynamic equation of the last inertia:

$$J_f a_f = J_f \dot{\omega}_f = d_{f-1}\Delta\omega_{f-1} + c_{f-1}\Delta\varphi_{f-1}.$$

It follows for output y:

$$y = \Delta\varphi_{f-1} = \frac{1}{c_{f-1}}\left(J_f\dot{\omega}_f - d_{f-1}\Delta\omega_{f-1}\right).$$

In the undamped case the output is described by

$$y = \Delta\varphi_{f-1} = \frac{J_f}{c_{f-1}}\dot{\omega}_f \tag{16}$$

and therefore the flat output describes the acceleration of the powertrain wheels. The acceleration of the powertrain wheels are a significant output to describe the drive quality.

Remark 3: A system representation with state

$$x = \begin{bmatrix} \varphi_1 & \varphi_2 & \dots & \varphi_f & \omega_1 & \omega_2 & \dots & \omega_f \end{bmatrix}^T$$

and output $y = \Delta\varphi_{f-1} = \varphi_{f-1} - \varphi_f$ is not a flat system in general. For example, consider a reduced undamped powertrain model with two inertias

$$\begin{aligned} \dot{\varphi}_1 &= \omega_1, \\ \dot{\varphi}_2 &= \omega_2, \\ \dot{\omega}_1 &= \frac{1}{J_1}\left[c_1(\varphi_1 - \varphi_2) - u\right], \\ \dot{\omega}_2 &= \frac{1}{J_2}\left[-c_1(\varphi_1 - \varphi_2)\right] \end{aligned}$$

and output $y = \varphi_1 - \varphi_2$. This system has relative degree $r = 2$, which is not equal to the system order $n = 4$.

2) Feedforward Control Law: The last equation of (11) is used to parametrize the input u. The parametrization of the input u leads to the feedforward control law.

Proposition 3: For the undamped system (Σ_0) from (10), define the feedforward control law

$$u_{FW} = a_1\left(y_{ref}^{(n)} - a_2 T^{-1} x_{ref}^*\right), \tag{17}$$

with

$$a_1 = \prod_{i=1}^{f-2}\frac{J_i}{c_i}J_{f-1}, \quad a_1 \in \mathbb{R}$$

and

$$a_2 = \left(K_{f-1,1}^{f-1}, \quad K_{f-1,2}^{f-1}, \quad \dots \quad K_{f-1,f-1}^{f-1}\right).$$

When u_{FW} is supplied as the feedforward input, the system trajectory of system (Σ_0) satisfies

$$x_{ref}^* = \begin{bmatrix} y_{ref} \; \dot{y}_{ref} \; \ddot{y}_{ref} \; \dots \; y_{ref}^{(n-1)} \end{bmatrix}^T.$$

Proof: The n^{th}-derivative of the flat output is [13]:

$$y^{(n)} = c^T A_0^n T^{-1} x^* + c^T A_0^{n-1} B u.$$

Solving for u yields

$$u = \left(c^T A_0^{n-1} B\right)^{-1}\left(y^{(n)} - c^T A_0^n T^{-1} x^*\right). \tag{18}$$

Applying this input as the system input for system (Σ_0) from (10), the system trajectories follow x^*, since (18) is the parametrization of the input u as a function of $y^{(n)}$ and x^*. By substituting the expressions for A_0^{n-1}, A_0^n with (14), (15) in (18) and using the chain structure of the stiffness matrix K from (3) yields the feedforward control law (17). ∎

Remark 4: The given feedforward control law needs only the stiffness matrix K with dimension $(f-1)\times(f-1)$ and not the system matrix A_0 with dimension $(2f-2)\times(2f-2)$.

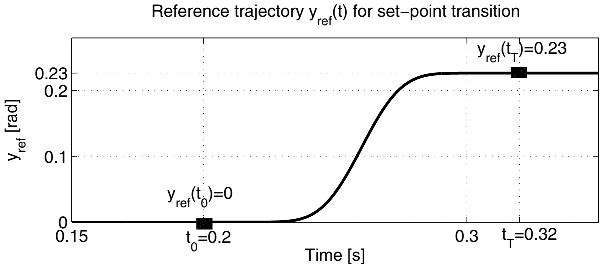

Fig. 4: A suitable 12 times differentiable polynomial reference signal for a set-point transition starting at time $t_0 = 0.2s$, with transition time $T = 0.12s$ from start point $y_{ref}(t_0) = 0\ rad$ to end point $y_{ref}(t_T) = 0.23\ rad$.

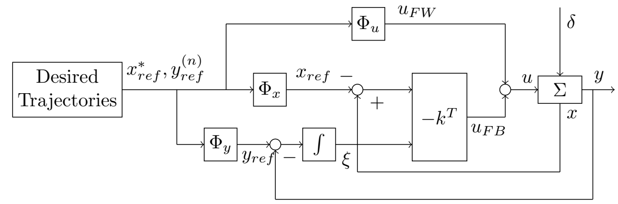

Fig. 5: Two-Degree-of-Freedom control structure with flatness based feedforward controller u_{FW} and feedback LQI controller u_{FB}. The maps Φ_x, Φ_u and Φ_y transform from the desired trajectories $(y_{ref}, \dot{y}_{ref}, \ddot{y}_{ref}, \ldots, y^{(n)}_{ref})$ to u_{FW}, x_{ref} and y_{ref}.

B. Trajectory Planning

As a tip-in maneuver is equivalent to a set-point transition, a desired trajectory with its *n* derivatives is needed for the control law (17) to steer from one equilibrium to another. The desired trajectory has to fulfill the following conditions:

- *n* times differentiable, since the *n* derivatives of y are needed for the control law,
- start point $y_{ref}(t_0) = y_{ref,0}$ and end point $y_{ref}(t_T) = y_{ref,T}$ and
- start and end point are in steady-state $y^{(i)}_{ref}(t)\Big|_{t=\{t_0,t_T\}} = 0, \quad i = 1, 2, \ldots, n.$

The start and end point of the reference output can be given as a function of the initial vehicle acceleration a_0 and a desired new vehicle acceleration a_T. The vehicle acceleration a is a translational dimension. It can be converted with the wheel radius r and by considering the total gear ratio $i_{gear} = ig_1 ig_2$ to an angular acceleration:

$$\dot{\omega}_f = \frac{a i_{gear}}{r_{wheel}}. \tag{19}$$

Using (19) in (16) yields the start and end point of the flat output:

$$\begin{aligned} y_{ref,0} =& \frac{1}{c_{f-1}} J_f \frac{a_0 i_{gear}}{r_{wheel}}, \\ y_{ref,T} =& \frac{1}{c_{f-1}} J_f \frac{a_T i_{gear}}{r_{wheel}}. \end{aligned} \tag{20}$$

Any trajectory, fulfills the above conditions is valid. For example, a polynomial trajectory can be chosen. Fig. 4 shows a twelve times differentiable polynomial reference trajectory for a powertrain model with seven inertias. This trajectory will be used in our simulation study.

Remark 5: The reference trajectory for different desired acceleration changes $\Delta a = a_{ref,T} - a_{ref,0}$ can be planned ahead and saved in tables.

C. Feedback Controller

In the two-degree-of-freedom control design structure, a linear-quadratic integral (LQI) controller is applied to deal with model mismatches and constant disturbances. We assume that the system state is known, either by measurements or a state observer. The LQI feedback control law is

$$u_{FB} = -k_1^T (x - x_{ref}) - k_2 \xi$$

with integrator output-error

$$\xi = \int_0^t (y_{ref}(\tau) - y(\tau))\, d\tau.$$

The feedback control law minimizes the cost function

$$J(u_{FB}) = \int_0^\infty \left(x_{ext}^T Q x_{ext} + u_{FB}^T R u_{FB} \right) dt,$$

with extended state $x_{ext} = \begin{bmatrix} x - x_{ref} \\ \xi \end{bmatrix}$ and positive semi-definite weight matrix $Q \in \mathbb{R}^{(n+1)\times(n+1)}$ and positive $r \in \mathbb{R}$.

The feedback controller as depicted in Fig. 5 is

$$u_{FB} = -k^T x_{ext} = -\begin{bmatrix} k_1^T, & k_2 \end{bmatrix} x_{ext}. \tag{21}$$

The system input of the overall control structure is the engine torque

$$T_{eng} = u = u_{FW} + u_{FB}.$$

IV. Simulation Results

We evaluate our two-degree-of freedom control approach on a powertrain simulation model via Matlab. The simulation model represent a powertrain with seven inertias as shown in Fig. 1. First, we evaluate the derived feedforward controller, then we compare our two-degree-of-freedom control approach to a feedback control approach with a static prefilter.

The system order of the simulation model is $n = 2f - 2 = 12$. In the simulation, the driver requests an acceleration change at time $t_0 = 0.2s$ with transition time $T = 120ms$. The demand is to change the acceleration from $a_0 = 0$ to $a_T = 3\frac{m}{s^2}$, which implies for the flat output with (20):

$$y_{ref,0} = 0, \quad y_{ref,T} = \frac{1}{c_6} J_7 \frac{a_T i_{gear}}{r_{wheel}}.$$

A twelve times differentiable polynomial reference trajectory as shown in Fig. 4 is generated for the request. In the simulation, the engine torque is the sum of the feedforward

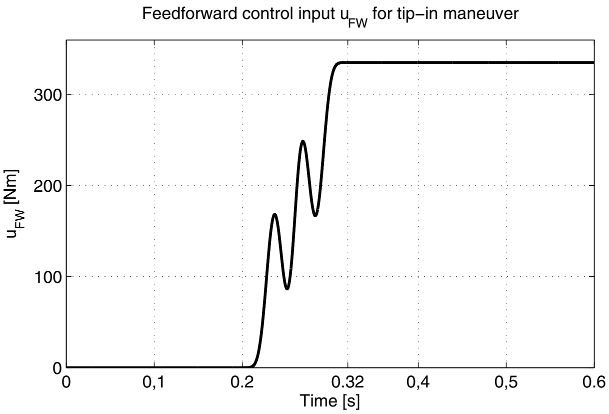

Fig. 6: Input u_{FW} generated by the flatness-based feedforward control law. The input trajectory decreases two times during the transition time to prevent driveline oscillation.

controller u_{FW} calculated in (17) and the feedback controller u_{FB} calculated in (21). Load disturbance is simulated as a constant. Sensor noise is not considered in our simulation, since it would not influence the feedforward controller.

For the described scenario, the feedforward control trajectory u_{FW} is depicted in Fig. 6. During the transition time between $0.2s$ and $0.32s$ two local maximums can be recognized. At the beginning of the acceleration the first inertias tend to have a higher acceleration than the last ones, since the first inertias are influenced earlier by the input. Unequal inertia accelerations lead to oscillation. The feedforward control input prevents unequal inertia accelerations by reducing the input torque in short time intervals. During this time the last inertias can 'catch up' to the acceleration of the first ones.

The performance of the feedforward controller for varying damper constants is shown in Fig. 7. We vary the nominal damper vector $d \in \mathbb{R}^{f-1}$ by doubling $2d$ and tripling $3d$. In the undamped case $d = 0$, the system output tracks the reference trajectory exactly, since the feedforward controller is designed based on an undamped system model. The tracking performance deteriorates for larger damper constants, especially after $0.3s$, whereas the transient behavior is very accurate. Even if the feedforward controller provides inexact tracking, the results are still close to the desired trajectory.

Fig. 8 and Fig. 9 compare the system output of our proposed two-degree-of-freedom controller with the system output of a LQI controller combined with a static prefilter. The static prefilter ensures that the LQI controller can reach a steady-state unequal to zero, but it does not consider the transient behavior of the powertrain system. Both methods have the same LQI weighting matrices Q and R. The matrices are chosen in such a way that the closed loop system is stable in the presence of a feedback time delay of $t_{delay} = 5ms$.

Fig. 8 shows the system output of both methods with an underdamped powertrain model without disturbances. The feedforward controller provides fast command response

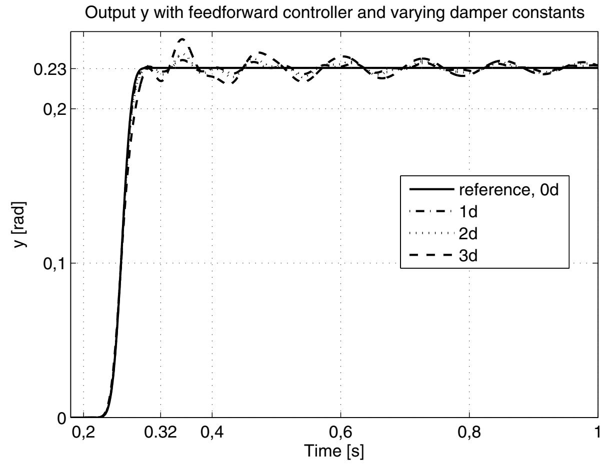

Fig. 7: Output y with feedforward controller for varying damper constants. No feedback controller and disturbances are added to the control loop.

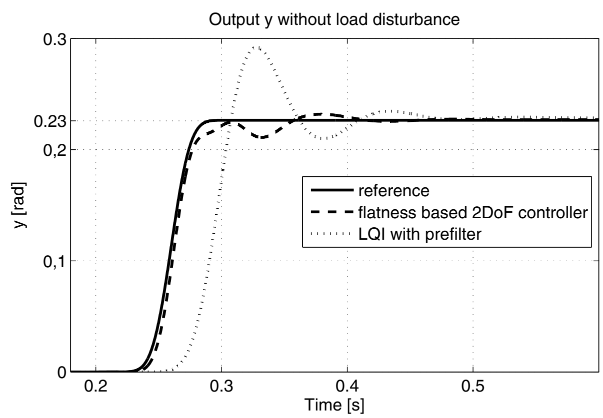

Fig. 8: Output of the proposed flatness based two-degree-of-freedom control approach in comparison with the output of the LQI controller approach with static prefilter without disturbances.

and accurate transient behavior in comparison to the LQI controller with prefilter. The system output of our proposed method has a fast tracking performance due to the feedforward controller. Furthermore, it does not overshoot in comparison to the LQI controller with prefilter. The feedforward controller can prevent overshooting, since it can keep the tracking error small during the transition time.

According to Fig. 9 both methods reach zero steady-state error in the presence of a constant disturbance of $100Nm$. The integrator of the feedback controller both methods ensures that the output error $e(t) = y_{ref}(t) - y(t)$ goes to zero for $t \to \infty$. The output error of the LQI regulator with prefilter is much larger than the error of the flatness based two-degree-of-freedom controller during the transient period.

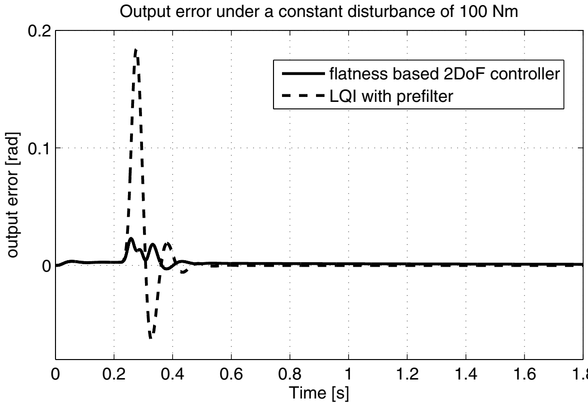

Fig. 9: Output error of the proposed flatness based two-degree-of-freedom controller in comparison with the output error of the LQI controller with static prefilter under constant load disturbance.

V. Conclusion and Future Work

In order to damp driveline oscillation caused by a tip-in maneuver, a two-degree-of-freedom controller has been developed and evaluated in simulation studies. The presented two-degree-of-freedom controller consists of a feedforward controller based on linear differential flatness theory and a linear-quadratic integral controller.

The focus of this paper was on analytical feedforward control design. First, we formulated a general f-degree-of-freedom model of the powertrain dynamics, for which we showed that in the undamped case the difference phase of the last two powertrain inertias is a flat output. Then, we derived a feedforward control law to fulfill the tip-in maneuver. Due to the presence of constant load torque, which acts on the powertrain as a disturbance, a linear-quadratic integrator controller was applied to reach zero steady-state error.

Simulation results showed that our approach enables a tip-in maneuver with fewer driveline oscillations and lower over-shooting than a feedback controller with static prefilter alone. Since output measurements are often time delay affected in powertrain and the performance of the feedback controller is therefore limited, the applied feedforward controller is an ‘add-on’ to the feedback controller to improve tracking performance.

The presented feedforward controller is model-based and therefore the performance of the controller depends on the accuracy of the system model. In our future work, we plan to design the feedforward controller based on a nonlinear powertrain model.

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