

Analysis of High Impedance Faults Using Fractal Techniques

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Abstract – Phase currents and voltages in a distribution power system change with a certain degree of chaos when high impedance faults (HIFs) occur. This paper describes application of the concepts of fractal geometry to analyze chaotic properties of high impedance faults. Root-mean-square rather than instantaneous values of currents are used for characterization of temporal system behavior; this results in relatively short time-series available for analysis. An algorithm is presented for pattern recognition and detection of HIFs; it is based on techniques suited for analysis of relatively small data sets. Examples are given to illustrate the ability of this approach to discriminate between faults and other transients in a power system.

Keywords: High impedance fault, power lines, fractals.

I. INTRODUCTION

High impedance faults represent one of the most difficult protection problems on power distribution systems today [1]. By definition, a high impedance fault does not draw enough current to cause conventional protective devices (e.g. fuses, relays, etc.) to operate. One reason that high impedance faults fail to operate protective devices is that they fail to establish a permanent return path. Instead, the effective impedance constantly and randomly changes as the fault's arcing behavior melts conductor, displaces contact soil, etc. Because of this behavior, the fault current magnitude is very chaotic [2].

Like high impedance faults, many other physical systems exhibit chaotic temporal behavior, especially under abnormal conditions. Different concepts of fractal geometry have been employed in recent years for quantization of chaotic behavior of nonlinear systems [3]. Fractal dimensions, Lyapunov exponents, entropies, attractors, etc. have been used with different rates of success for distinction between random noise in the system and chaos determined by some underlying physical process [4].

Studies of time varying signals have been performed in many fields of engineering and science. Most algorithms and approaches, especially at the early stages of research, have been applied to data sets containing tens of thousands of data points. For such series, estimation of fractal dimensions can be performed quite reliably. In recent years, estimation of fractal dimensions has been sought for small data sets as well, with their length in the range of several hundred data points, for example [5]. However, even when low-dimensional chaos is analyzed, the reliability and correctness of estimation of fractal dimensions of small data sets is questionable, due to inherent properties of fractal dimension calculation algorithms [6].

This paper presents an approach for quantization of power system behavior which lies between strict fractal dimension calculation algorithms and statistical estimation of randomness of change of current values in the system. This compromise allows adjustment of sensitivity of the algorithm to changes of input values on different time scales, from fractions of a second to multiple minutes.

II. HIGH IMPEDANCE FAULT DATA

Texas A&M University's Power System Automation Laboratory operates a Downed Conductor Test Facility (DCTF). This affords the opportunity to stage downed conductor faults on an operating 12.47 kV multi-grounded wye circuit that serves several megawatts of load in the surrounding area. TAMU has used the DCTF to stage several hundred such faults over the past several years.

Conventional CTs and PTs at the substation enable measurement of feeder currents and voltages during tests at the DCTF. In addition an instrumentation building at the substation per-

mits installation of long term monitoring equipment. In this way, TAMU can study “normal” system behavior as well as fault behavior.

For several years, TAMU has had in place at the substation monitoring equipment that stores data whenever it sees abnormal system behavior. Because of the long records of interest (e.g. several minutes), it is not feasible to record high bandwidth data. Also, previous research shows that much information is contained in the RMS behavior of the system currents [7]. For this reason, the monitoring equipment calculates and stores RMS values at a rate of 30 values per second. The data contained in the following Figs. resulted from data from this monitor, both for normal system events and for fault data.

Although the proposed analysis has been applied to phase currents, similar techniques can be used for return conductor currents, as well as line voltages. Adjustment of the algorithm input parameters would be necessary to accommodate for different internal characteristics of physical values.

III. FRACTAL DIMENSIONS

A. Theoretical Background

The concept of fractal dimension has evolved from relatively simple intuitive ideas to elaborate algorithms during the last decade. Several fractal dimensions have been proposed as measures of chaotic system behavior. Some of the most commonly used (e.g. capacity, information, and correlation fractal dimensions), have been evaluated using recorded data of the currents in a power system and FD3 software based on techniques in [8]. Not going into details available from the cited references, we will briefly define these dimensions here.

Given the S set of N data points $\{\mathbf{R}_i\}$ in a metric space, a capacity fractal dimension can be found from (1), using the so-called box counting technique [9],

$$d_B = \lim_{\epsilon \rightarrow 0} \frac{\log N_B(\epsilon)}{\log(1/\epsilon)}, \quad (1)$$

where $N_B(\epsilon)$ is a minimal number of boxes (squares for two-dimensional sets) needed to cover the set, and ϵ is the linear size of the box.

Information dimension d_I is similar to the capacity dimension defined by (1), except that N_B is substituted with the average information entropy function [10]:

$$I(\epsilon) = \sum_{i=1}^{N(\epsilon)} -P(\epsilon, k) \log(P(\epsilon, k)), \quad (2)$$

where $P(\epsilon, k)$ is the probability of the event that the k -th box of size ϵ contains point of set S .

One of the most popular measures is the spatial correlation dimension [11],

$$d_C = \lim_{r \rightarrow 0} \frac{\log C(r)}{\log r}, \quad (3)$$

where the correlation integral, $C(r)$, is given by

$$C(r) = \lim_{N \rightarrow \infty} \frac{1}{N^2} \sum_{j=1}^N \sum_{i=j+1}^N H(r - \|\mathbf{R}_i - \mathbf{R}_j\|), \quad (4)$$

where H is the Heaviside function, and the norm is Euclidian.

The relationship between these three dimensions is defined by the inequality [12]:

$$d_C \leq d_I \leq d_B. \quad (5)$$

Although this inequality is theoretically strict, calculated values of fractal dimensions of different objects do not always conform to it, which may serve as an indicator of poor estimation of the fractal dimensions.

B. Calculation of fractal dimensions

The temporal dynamics of power system currents is normally low-dimensional, with values of fractal dimension varying between 1 and 2. This has been confirmed by calculation of fractal dimensions for time-series data sets for both normal and faulted conditions. Does the faulted line exhibit significantly different fractal properties than under normal conditions, and can this feature be used for detection and discrimination of high impedance faults? There is no single answer to this question. While fractal characteristics of chaotic behavior caused by HIFs are different from normal operation values, the direct use of fractal dimensions for pattern recognition purposes seems to be unreasonable.

Fig. 1 shows the RMS data from a downed conductor fault on grass. Table 1 illustrates results of calculation of fractal dimensions for the data set shown in Fig. 1. The data set consists of 2048 data points. The set was partitioned into eight subsets of 256 data points each, and fractal dimensions have been calculated separately.

The absolute theoretical minimum number of data points needed for estimation of fractal dimension is 10^d [8], which is about 40 points for the values in Table 1. Another sometimes recommended estimate of minimum number of data points is 2^{4d} , which results in 60 data points. Although these numbers are below 256, several indications are available to demonstrate expected poor performance of 256-point estimation.

One can immediately notice from Table 1 that while estimation for the whole set conforms to (5), the values of fractal dimensions for the subsets jump around and sometimes exhibit a symmetrically opposite inequality property. The values for periods which include transition from normal to fault state and back are unrealistically low. Considering the variance of the values in each column, they can hardly be used directly for fault detection. In order to be more stable, such an approach would require significantly longer subsets than 256 points, which would make the whole detection process inherently slow.

The traditional box-counting technique used for estimation of fractal dimension involves monotone increase of the size of

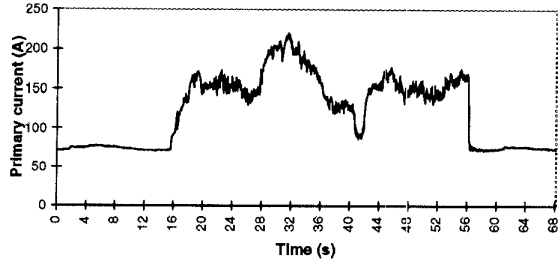


Fig. 1: Staged fault with 1/0 Al conductor on grass.

the time-current rectangles (boxes). In our case, this procedure of increasing the size of the boxes quantizes chaotic temporal variations on different time scales. A somewhat similar, but more flexible concept of combined use of fractal number and a classical smoothing function is proposed next.

As an aside, it should be noted here that more precise estimation of fractal dimension of current signals could be achieved by using larger sets of instantaneous values of currents. However, the computational time t_c required for calculation of fractal dimension by the algorithm increases significantly with the number of data points N_B in the subset[8],

$$t_c = f(d_B)N_B \log N_B, \quad (6)$$

where $f(d_B)$ is a slowly varying function which depends on hardware and software used. This time increase hinders the possibility of real-time on-line fault identification. Only RMS values have been used in this paper. Nevertheless, use of more effective algorithms for estimation of fractal dimension could lead to the use of instantaneous current values in the future.

IV. DESCRIPTION OF THE ALGORITHM

One of the major difficulties in the detection of HIFs is that the amplitude of the current drawn by the fault may constitute only a small percentage of the total line current. Thus, the total current is not high enough to operate conventional protection. In order to address this problem, the detection algorithm has been designed to be insensitive to the amplitude changes of the currents, unless their chaotic temporal behavior changes with the amplitude as well. This is achieved by normalization of the current values in each data subset so that they are in the range $[0, 1]$. Since the input time scale is initially given in seconds, it is scaled down by a factor of 100, so that the Euclidian metric is sensitive to changes along both time and current axes.

The algorithm for real-time detection of HIFs is shown in Fig. 2. A sliding time window is used for calculations of various parameters described below.

A. Fractal Number

For our purposes, fractal number F of the m -th data set S_m which contains N data points is defined as

$$F = \left[1 - \frac{\sum_{j=1}^{N-l} \|\mathbf{R}_{j+k} - \mathbf{R}_j\|}{\sum_{j=1}^{N-l} \|\mathbf{R}_{j+l} - \mathbf{R}_j\|} \right] \cdot 100, \quad (7)$$

TABLE 1: FRACTAL DIMENSIONS FOR FIG. 1.

Time (s)	Fractal Dimension		
	Capacity	Information	Correlation
0-8.5	1.239	1.180	1.141
8.5-17	0.911	0.961	0.952
17-25.5	1.199	1.274	1.235
25.5-34	1.160	1.118	1.082
34-42.5	1.137	1.088	1.066
42.5-51	1.209	1.213	1.213
51-59.5	0.970	0.973	0.878
59.5-68	1.335	1.476	1.458
0-68	1.257	1.207	1.068

where k and l are small integers, which define the time interval between neighboring data points, $l > k$. Different norms can be assumed, however, a conventional Euclidian norm has been chosen. As can be seen from (7), the ratio of the metric lengths of the same time sequence is found using two different sampling rates. The algorithm was found to be more effective when the ratio l/k was not an integer. In this case, not all data points from the l -step sampled subset belong to the k -sampled subset, which sometimes causes the fractal number to be less than zero. The minimum sampling frequency required for this algorithm is defined by the smallest common divisor of l and k .

B. Heaviside Function

The Heaviside function $H(s)$ of a parameter s is normally defined as:

$$\begin{aligned} H(s) &= 1 \text{ if } s \geq 0 \\ H(s) &= 0 \text{ if } s < 0. \end{aligned} \quad (8)$$

Once the fractal number, F , is found for the subset of length N , its value is compared with a predetermined threshold value T . If the chaotic variations of the time series are relatively low, and no spikes are present in the subset, the fractal number F is lower than the threshold value T , and the Heaviside function is set to 0. If the Heaviside function is higher than the threshold value, it is set to 1, as shown in Fig. 2:

$$s(t) = |F| - T. \quad (9)$$

The period between changes of the Heaviside function is N times as long as the sampling period. This procedure is similar to increasing the box size in the box-counting algorithm.

C. Smoothing Function

Chaotic variations of the fractal number are not always above the threshold level during the fault, and neither are they always below it during normal conditions. However, they are

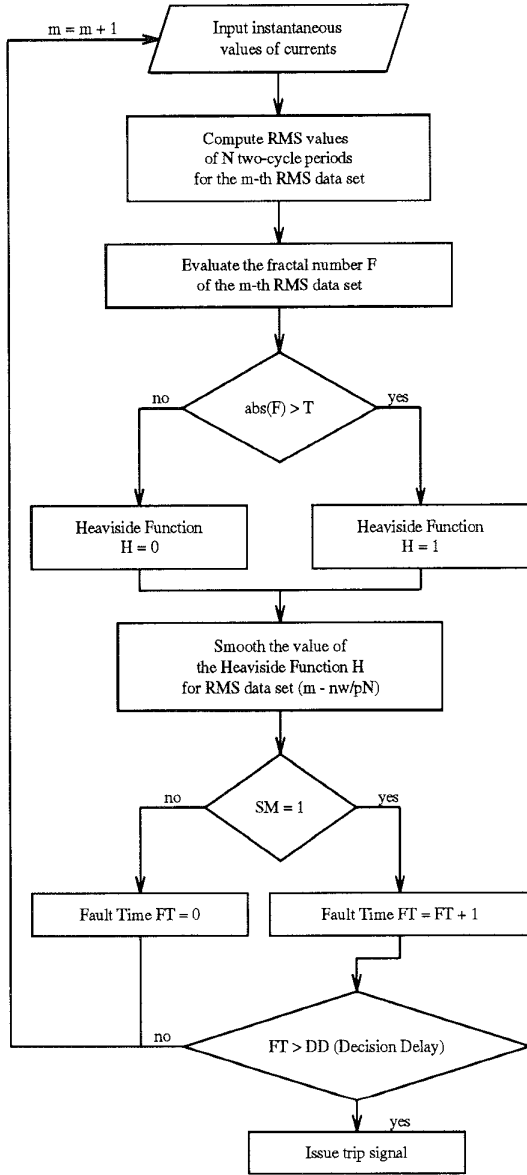


Fig. 2: Fault identification algorithm.

generally higher during the fault. The decision-making mechanism requires transition from flickering values of the Heaviside function to the state of accumulated confidence that the high-impedance fault indeed has happened, and it has not self-cleared. This is achieved by using a smoothing technique commonly used in image processing.

Smoothing function $SM(t)$ for the time-changing Heaviside function $H(s(t))$ at the time t_0 is defined here as

$$SM(t_0) = 1 \text{ if } \int_{t_0-w/2}^{t_0+w/2} H(s(t))dt \geq 0.5$$

$$SM(t_0) = 0 \text{ if } \int_{t_0-w/2}^{t_0+w/2} H(s(t))dt < 0.5. \quad (10)$$

where w is the width of the smoothing window. In the given examples, $w = 7N$. For better performance, the smoothing has been performed twice in the examples below, by putting $SM(t)$ under the integral instead of $H(s(t))$ in the second run of (10).

D. Fault Identification

The final decision for identification of the fault, whether used directly for tripping a circuit breaker or in combination with other detection algorithms is done after the output of the smoothing function has been equal to 1 for a predetermined period of time. The time p between two consecutive data points in the set S_m before scaling of the time axis is 1/30 seconds. For $N = 20$ in the following examples, the real-time duration of each data subset S_m is 0.667 seconds. An intrinsic delay of $nw/2$ in the decision making process exists where n is the number of runs of a smoothing function. The shaded rectangles in Figs. 3d, 4d, and 5d show the width of the sliding window which contains information necessary for the output of the fault identification function. Half of the time-side of each rectangle corresponds to the minimum Decision Delay in Fig. 2.

V. DEMONSTRATION OF PERFORMANCE

Fig. 3a shows the RMS current measured at the substation during a fault in which a downed 7.2 kV conductor was on grass. The load current on the system was approximately 60 A, with a superimposed fault component ranging up to 100 A. Parts b - d of Fig. 3 show that the algorithm correctly identified the fault approximately four seconds after it started. This range of time is acceptable for low current faults at the distribution level (5-35 kV). In fact, it is even desirable so that conventional low level devices such as fuses can operate on a more localized basis. Note that the algorithm indicates continuously for nearly one minute. Then it begins to fail as the ground dries and glassifies, resulting in diminishing fault current.

The problem encountered with existing algorithms is that not only faults, but also different transients are sometimes detected, and distinguishing between transients and faults is hard to perform automatically. Properties of the described approach are illustrated in Fig. 4 and Fig. 5. Note, however, the different dependent axis ranges in these two Figs. and Fig. 3. Fig. 4 shows the results when applied to a normal system serving several oilfield pump jacks; these pump jacks cause cyclical variations in current flow based on their mechanical pumping cycle, several seconds per cycle. Sharp transients in Fig. 5 probably are caused by unsuccessful starting of a large electric motor; the Heaviside function is virtually unaffected by them because the values of the fractal number remain low.

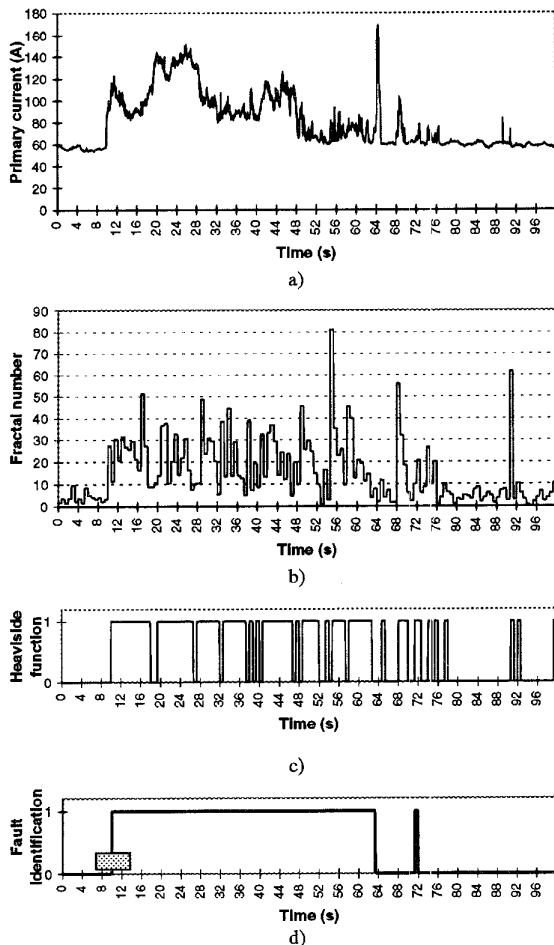


Fig. 3: Staged fault with 1/0 Al conductor on grass.

One random spike, still of a fairly low value is picked up by the Heaviside function but eliminated in the smoothing step.

VI. ACKNOWLEDGMENTS

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VII. CONCLUSIONS

High impedance faults on power distribution systems remain a perplexing problem. Although prior research has provided hope for the detection of many of these faults, there remains room for improvement.

A signature of high impedance faults can be detected in the time series of changing RMS values of power line currents using concepts of fractal analysis. Direct calculation of fractal dimensions is not effective due to relatively short data

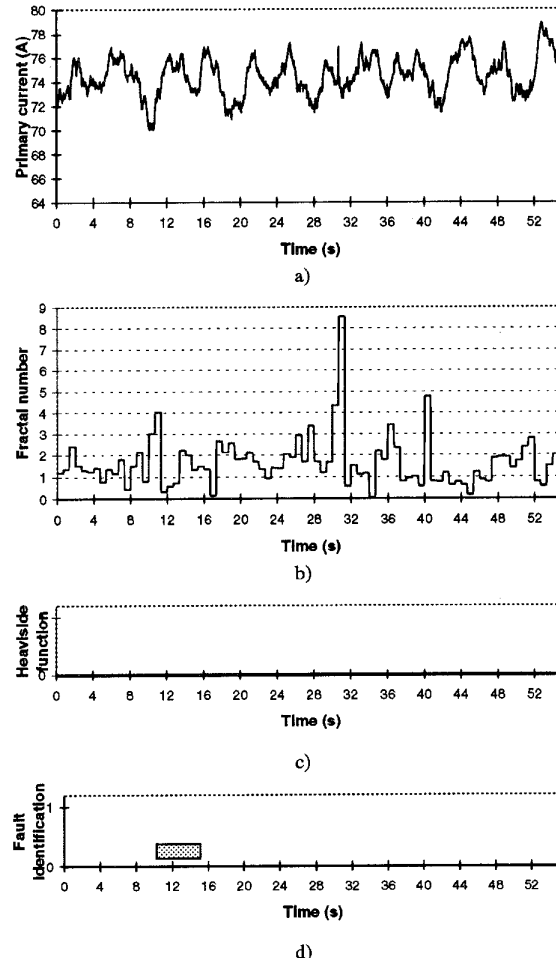


Fig. 4: Normal system operation.

sets available for estimation. The described algorithm allows distinction between switching transients in the power system and faults. Sensitivity to different aspects of chaotic changes in time series can be adjusted by changing several key parameters of the algorithm.

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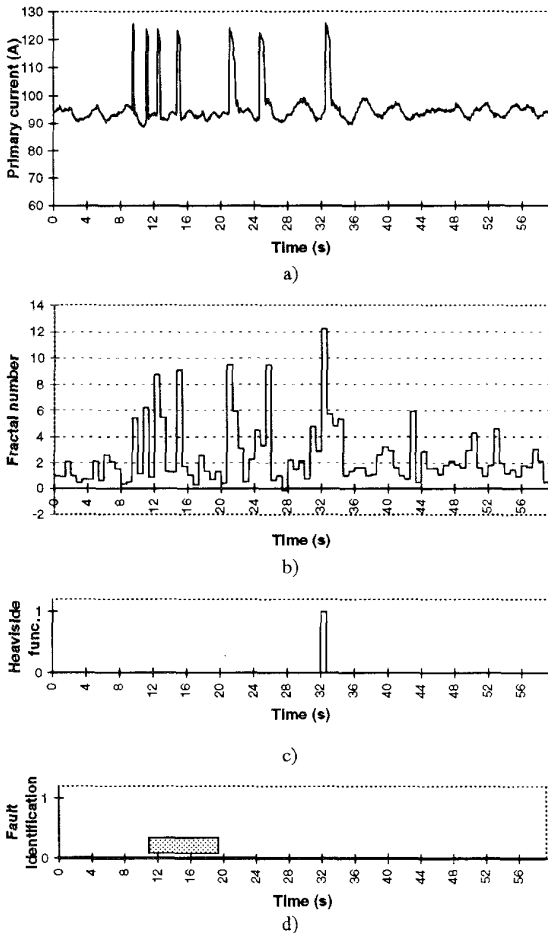


Fig. 5: Transients.

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