Topics: 1. Matrix treatment of polarization (Chapter 14)
2. Reflection and refraction at dielectric interfaces
   - Fresnel equations (Sec. 23.1 – 23.5)
3. Polarization phenomena and devices
   (Sec. 15.1, 15.2, 15.4–15.6, 17.5)

* Polarization of Light

Polarization: Time trajectory of the end point of the E-field direction, facing where the wave is coming from

\[ \mathbf{E} = E_x \hat{x} + E_y \hat{y} \]
\[ \mathbf{E}_x = E_{0x} e^{i(kz - wt + \phi_x)} \]
\[ \mathbf{E}_y = E_{0y} e^{i(kz - wt + \phi_y)} \]
\[ \mathbf{E} = (E_{0x} e^{i\phi_x} + y E_{0y} e^{i\phi_y}) e^{i(kz - wt)} = \mathbf{E}_0 e^{i(kz - wt)} \]

* Matrix Representation – Jones Vectors

\[ \mathbf{E}_0 = \begin{bmatrix} E_{0x} e^{i\phi_x} \\ E_{0y} e^{i\phi_y} \end{bmatrix} \]
\[ \sqrt{E_{0x}^2 + E_{0y}^2} = 1. \]

- Linearly polarized light:

  \[ \mathbf{E}_x = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \]
  \[ \mathbf{E}_y = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \]

  \[ \mathbf{E}_x = \begin{bmatrix} \cos \alpha \\ \sin \alpha \end{bmatrix} \]

- Left-circular polarization (LCP):

  \[ E_x = A \cos wt. \]
  \[ \mathbf{E}_x = E_{0x} e^{iwt} \]

  \[ E_y = A \sin wt = A \cos (wt - \frac{\pi}{2}) \]
  \[ E_y = E_{0y} e^{-iwt} \]

  \[ \phi = \phi_0 - \phi_0 \]
\[ E_y = A \sin \omega t = A \cos(\omega t - \frac{\pi}{2}) \quad \therefore \quad \hat{E}_y = E_{oy} e^{-i(\omega t - \frac{\pi}{2})} \]

\[ \epsilon = \phi_y - \phi_x \]

\[ \epsilon = \phi_y = \frac{\pi}{2} - \phi_x = 0 \quad E_{ox} = E_{oy} = A \]

\[ \hat{E}_o = \left[ \begin{array}{c} A e^{i\frac{\pi}{2}} \\ A e^{-i\frac{\pi}{2}} \end{array} \right] = A \left[ \begin{array}{c} 1 \\ i \end{array} \right] \text{ Normalize } \frac{1}{\sqrt{2}} \left[ \begin{array}{c} 1 \\ i \end{array} \right] \]

- **Right-circular polarization (RCP):**

\[ \hat{E}_o = \frac{1}{\sqrt{2}} \left[ \begin{array}{c} 1 \\ i \end{array} \right] \]

* *Elliptical Polarization*

\[ \hat{E}_o = \frac{1}{\sqrt{A^2 + B^2}} \left[ \begin{array}{c} A \\ iB \end{array} \right] \quad \phi_y - \phi_x = \frac{\pi}{2} \text{ or } -\frac{\pi}{2} \]

- General case,

\[ \hat{E}_o = \left[ \begin{array}{c} E_{ox} e^{i\phi_x} \\ E_{oy} e^{i\phi_y} \end{array} \right] = \left[ \begin{array}{c} A e^{i\epsilon} \\ b e^{i\epsilon} \end{array} \right] \]

\[ b e^{i\epsilon} = b (\cos \epsilon + i\sin \epsilon) = B + ic \]

\[ \hat{E}_o = \left[ \begin{array}{c} A \\ B + ic \end{array} \right] \text{ Normalize } \frac{1}{\sqrt{A^2 + B^2 + c^2}} \left[ \begin{array}{c} A \\ B + ic \end{array} \right] \]

- **Elliptical trajectory:**

\[ \epsilon = \tan^{-1} \frac{c}{B} \]

\[ \tan 2\alpha = \frac{2E_{ox} \overline{E_{oy} \cos \epsilon}}{E_{ox}^2 - E_{oy}^2} \]

\[ B = 0, \quad \epsilon = \pm \frac{\pi}{2}, \quad \alpha = 0 \rightarrow \bigcirc \]

\[ C = 0, \quad \epsilon = 0 \rightarrow \text{Linear polarization,} \]

Equation of the ellipse:

\[ \left( \frac{E_{ox}}{E_{ox}} \right)^2 + \left( \frac{E_{oy}}{E_{ox}} \right)^2 - 2 \left( \frac{E_{ox}}{E_{ox}} \right) \left( \frac{E_{oy}}{E_{oy}} \right) \cos \epsilon = \sin^2 \epsilon \]

* *Superposition of Waves*

→ Summation of Jones vectors.

*Example:*

\[ \left[ \begin{array}{c} 1 \\ i \end{array} \right] + \left[ \begin{array}{c} 1 \\ -i \end{array} \right] = \left[ \begin{array}{c} 2 \\ 0 \end{array} \right] \]

\[ \left[ \begin{array}{c} 1 \\ i \end{array} \right] + \left[ \begin{array}{c} 2 \\ 3 \end{array} \right] = \left[ \begin{array}{c} 3 \\ 4 \end{array} \right] \]
\[
\begin{bmatrix}
1
1
-1
1
\end{bmatrix}
+ \begin{bmatrix}
2
-1
i
\end{bmatrix}
= \begin{bmatrix}
3
-i
\end{bmatrix}
\]

\[
\begin{bmatrix}
1
1
\end{bmatrix}
+ \begin{bmatrix}
e^{in}
1
\end{bmatrix}
= \begin{bmatrix}
1
1
\end{bmatrix}
+ \begin{bmatrix}
-e^{in}
1
\end{bmatrix}
= \begin{bmatrix}
2
i
\end{bmatrix}
\]

* **Jones Matrix**
  (for polarization devices)

\[
M = \begin{bmatrix}
a & b \\
c & d
\end{bmatrix}
\]

\[
\text{Jones Vector} \xrightarrow{\text{Polarization device}} \text{Out} = M \cdot \text{In}
\]

* **Linear Polarizer**
  - Vertical (y) transmission axis (TA)
    
    For all Jones vector \[
    \begin{bmatrix}
x \\
y
\end{bmatrix},
    \begin{bmatrix}
a & b \\
c & d
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
= \begin{bmatrix}
y \\
y
\end{bmatrix}
\]
    
    \[
    ax + by = 0 \quad \text{for all } x, y \rightarrow \begin{bmatrix}
a & b \\
c & d
\end{bmatrix} = \begin{bmatrix}
0 & 0 \\
0 & 1
\end{bmatrix}
\]

  - Horizontal (x) transmission axis \[
  \rightarrow \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\]

  - Linear polarizer with TA at angle \(\theta\)
    \[
    M = \begin{bmatrix}
    \cos^2 \theta & \sin \theta \cos \theta \\
    \sin \theta \cos \theta & \sin^2 \theta
    \end{bmatrix}
    \]

* **Phase Retarder (Wave Plate)**
  - \(\Delta \phi = 90^\circ\): Quarter-wave plate (QWP)
  - \(\Delta \phi = 180^\circ\): Half-wave plate (HWP)

\[
E_x e^{i\phi_x} \rightarrow E_x e^{i(\phi_x + \phi_x)}
\]

\[
E_y e^{i\phi_y} \rightarrow E_y e^{i(\phi_y + \phi_y)}
\]

\[
\begin{bmatrix}
e^{i\phi_x}
0
0
e^{i\phi_y}
\end{bmatrix}
\begin{bmatrix}
E_x e^{i\phi_x} \\
E_y e^{i\phi_y}
\end{bmatrix}
= \begin{bmatrix}
E_x e^{i(\phi_x + \phi_x)} \\
E_y e^{i(\phi_y + \phi_y)}
\end{bmatrix}
\]

\[
\text{phase retarder} \xrightarrow{\text{Jones matrix}} \text{QWP with y-axis as SA}.
\]
\[ \varepsilon_y - \varepsilon_x = \frac{\pi}{2} \rightarrow \varepsilon_y = \frac{\pi}{4}, \varepsilon_x = -\frac{\pi}{4} \]

\[ \therefore M = \begin{bmatrix} e^{-i\frac{\pi}{4}} & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{bmatrix} = e^{-i\frac{\pi}{4}} \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \quad "\"\text{for x axis as SA.}\]

\[ \text{HWP} \quad M = \begin{bmatrix} e^{-i\frac{\pi}{2}} & 0 \\ 0 & e^{i\frac{\pi}{2}} \end{bmatrix} = e^{-i\frac{\pi}{2}} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \]