EE485 Introduction to Photonics

Superposition of Waves and Interference

1. Two-beam interference and interferometry
2. Multi-wave interference
3. Fabry-Perot interferometer
4. Group/phase velocity and dispersion

Reading: Pedrotti, Sec. 5.1-5.2, 5.5-5.6, 7.1-7.2, 7.4, 7.9, 8.1-8.7, 8-9, 10.7-10.8
Superposition Principle

\[ \psi_1(\mathbf{r}, t), \quad I_1 \]

\[ \psi_2(\mathbf{r}, t), \quad I_2 \]

\[ \sqrt{\psi(\mathbf{r}, t) = \psi_1(\mathbf{r}, t) + \psi_2(\mathbf{r}, t)} \]

or \[ X \quad I = I_1 + I_2 \]

or \[ X \quad \text{Both?} \]

Formally speaking,

If \( \psi_1 \) and \( \psi_2 \) are independently solutions of the wave equation,

\[ \nabla^2 \psi = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} \]

then the *linear combination*,

\[ \psi = a\psi_1 + b\psi_2 \]

\[ a, b : \text{constants} \]

is also a solution.

For electromagnetic waves,

\[ \mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 \quad \mathbf{H} = \mathbf{H}_1 + \mathbf{H}_2 \]

(Orientation of the fields must be taken into account.)
Two-Beam Interference

\[ E_1 = E_{01} \exp[i(k_1 \cdot r - \omega t + \varepsilon_1)] \equiv E_{01} \exp[i(\phi_1 - \omega t)] \]
\[ E_2 = E_{02} \exp[i(k_2 \cdot r - \omega t + \varepsilon_2)] \equiv E_{02} \exp[i(\phi_2 - \omega t)] \]
\[ E = E_1 + E_2 \]

Recall from Light as Electromagnetic Waves, optical intensity =

\[ I = \langle |S| \rangle = \left| \text{Re}\left\{ \frac{1}{2} E(r) \times H(r)^* \right\} \right| \propto |E(r)|^2 \]

For the combined wave,

\[ I \propto (E_1(r) + E_2(r)) \cdot (E_1^*(r) + E_2^*(r)) \]

If \( E_{01} \) and \( E_{02} \) are parallel to each other,

\[ I = I_1 + I_2 + \sqrt{I_1} \sqrt{I_2} \exp[i(\phi_1 - \phi_2)] + \sqrt{I_1} \sqrt{I_2} \exp[i(\phi_2 - \phi_1)] \]

\[ \rightarrow I = I_1 + I_2 + 2 \sqrt{I_1 I_2} \cos \delta \]
\[ \delta = \phi_1 - \phi_2 \]

Fringe contrast = \[ \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} \]

![Figure 10-1 Irradiance of interference fringes as a function of phase. Fringe contrast is enhanced in (b), where the background irradiance \( I_{\text{background}} = 0 \) when \( I_1 = I_2 \).](image)
Young’s Double-slit Experiment

\[ \phi = k \alpha \sin \theta \]
\[ I = 2I_0 [1 + \cos(k \alpha \sin \theta)] \]
\[ = 4I_0 \cos^2 \left( \frac{\pi \alpha \sin \theta}{\lambda} \right) \]

For points P near the optical axis,
\[ I = 4I_0 \cos^2 \left( \frac{\pi ay}{\lambda L} \right) \]

Discussion
If a thin plate of glass is placed over one of the slits, what will happen to the fringe pattern?
Michelson Interferometer (I)

\[ I = 2I_0 \left[ 1 + \cos \left( 2\pi \frac{\Delta}{\lambda} \right) \right] \]

\( \Delta \): Optical path length difference

Dark fringes: \( 2d \cos \theta = m\lambda \)

\[ |\Delta \theta| = \frac{\lambda \Delta m}{2d \sin \theta} \]
Michelson Interferometer (II)

**Sensing small refractive index change**

Start with dark fringe in the center,

\[
I(n) = 2I_0 \left[ 1 + \cos \left( 2\pi \frac{(n-1)2x}{\lambda} + \pi \right) \right]
\]

Example:

\[
\lambda = 632.8 \text{ nm}
\]
\[
x = 0.5 \cdot 10^5 \cdot \lambda = 3.164 \text{ mm}
\]

→Δn of 10⁻⁵ results in 2π phase change.

**Resolving small wavelength difference (Spectrometer)**

Start with coincident bright fringe in the center for both \(\lambda_1\) and \(\lambda_2\).

Move the mirror by \(\Delta d\) until the next coincidence occurs.

\[
\Delta \lambda = \frac{\lambda^2}{2\Delta d}
\]
Example: Design a fiber-optic Mach-Zehnder interferometer that can demultiplex two light signals of free-space wavelength $\lambda_1 = 1551$ nm and $\lambda_2 = 1550$ nm. Through which output port would light of $\lambda_3 = 1549$ nm and $\lambda_4 = 1548$ nm exit?
Interference in Dielectric Films

Additional $\pi$ phase shift occurs when reflected by higher index material.

In normal incidence (from $n_1$ material to $n_2$ material), the reflection coefficient

$$r = \frac{1 - n}{1 + n}$$

$$n = n_2 / n_1 : \text{Relative index}$$

Anti-reflection (AR) coating (0-th order):

$$n_f = \sqrt{n_0 n_s}, \quad t = \lambda_f / 4$$
Multi-Wave Interference
— Equal Amplitude and Equal Phase Difference

\[ U_m = \sqrt{I_o} \exp[i(m-1)\phi], \quad m = 1, 2, \ldots, M \]

\[ U = \sum_{m=1}^{M} U_m = \sqrt{I_o} \frac{1 - \exp(iM\phi)}{1 - \exp(i\phi)} \]

\[ I = |U|^2 = I_0 \frac{\sin^2(M\phi/2)}{\sin^2(\phi/2)} \]

Interesting features:

- Mean intensity

\[ \bar{I} = \frac{1}{2\pi} \int_{0}^{2\pi} II d\phi = MI_0 \]

- Peak intensity = \( M^2I_0 \)

- Intensity drops to zero at \( \phi = 2\pi/M \) from peak intensity.

- Sensitivity to \( \phi \) increases with \( M \).

- (\( M - 2 \)) minor peaks between major peaks.
Multi-Wave Interference
Example: Bragg Reflection

Intensity of the reflected light maximizes at

$$\sin \theta = \frac{n\lambda}{2d}$$

One of the applications:
Characterizing the lattice constant of a crystal.

$$\lambda = 1.16 \, \text{Å}$$

For (200) direction,

$$d = \frac{1.16}{2 \sin(27^\circ / 2)} = 2.48 \, \text{Å}$$
Multi-Wave Interference — Progressively Smaller Amplitude and Equal Phase Difference

Parallel plate

Fabry-Perot Interferometer

Two high-reflection plates separated by a distance $d$. $d$ is often tunable.

$r, r’$: Reflection coefficient
$t, t’$: Transmission coefficient
ρ₁, ρ₂: Reflection coefficients
t₁, t₂: Transmission coefficients
(These all work on fields.)

Consider just to the right of mirror M₁,

\[
\sum_{0}^{\infty} E_n^+ = \frac{E_0}{1 - \rho_1 \rho_2 e^{-i2\theta}}
\]

θ = \frac{2\pi}{\lambda_0} nd

Furthermore,

\[
E_0 = t_1 E_i
\]

\[
E_{tr} = t_2 e^{-i\theta} \sum E_n^+
\]
Fabry-Perot Interferometer — Spectra

Define power reflectivity of the mirror \( R = |\rho|^2 \), \( |t|^2 = 1 - R \)

Transmittance
\[
T = \left| \frac{E_{tr}}{E_i} \right|^2 = \frac{(1 - R_1) \cdot (1 - R_2)}{(1 - \sqrt{R_1 R_2})^2 + 4 R_1 R_2 \sin^2 \theta}
\]

Reflectance
\[
R_{net} = \left| \frac{E_r}{E_i} \right|^2 = 1 - T
\]

Quiz: For sharper transmission spectrum, do we want higher \( R_{1,2} \) or lower \( R_{1,2} \)?
Fabry-Perot Interferometer — Parameters

Free spectral range (FSR)

\[ \nu_{FSR} = \frac{c}{2nd}, \quad \lambda_{FSR} = \frac{\lambda_0^2}{c} \nu_{FSR} \]

FWHM

\[ \Delta \nu_{1/2} = \frac{c}{2nd} \frac{1 - \sqrt{R_1R_2}}{\pi(R_1R_2)^{1/4}} \]

Cavity Q-factor

\[ Q = \frac{\nu_0}{\Delta \nu_{1/2}} = \frac{2\pi nd}{\lambda_0} \frac{(R_1R_2)^{1/4}}{1 - \sqrt{R_1R_2}} \]

Finesse

\[ \tilde{\mathcal{F}} = \frac{\nu_{FSR}}{\Delta \nu_{1/2}} = \frac{\pi(R_1R_2)^{1/4}}{1 - \sqrt{R_1R_2}} \]

Example

A HeNe laser (\( \lambda = 632.8 \text{ nm} \)) cavity is defined by \( d \sim 1 \text{ m} \), \( R_1 = R_2 = 0.99 \)

\[ \nu_0 = 4.74 \times 10^{14}, \Delta \nu_{1/2} = 480 \text{ kHz}, \, Q = 9.88 \times 10^8, \]

and \( \tilde{\mathcal{F}} = 313. \)
Fabry-Perot Interferometer — Tunable Filter and Spectrometer

Resonant condition

\[ \theta = \frac{\omega}{c} d = \frac{2\pi nd}{\lambda_0} = m\pi \]

Resonant wavelength

\[ \lambda_0 = \frac{2nd}{m} \]

But the Fabry-Perot interferometer has finite passband width …

Resolving criterion:
Spacing less than what’s defined by \( T = 0.5 T_{\text{max}} \)

\[ \Delta\lambda_{\text{min}} = \frac{\lambda_0^2}{2nd} \frac{1 - \sqrt{R_1R_2}}{\pi(R_1R_2)^{1/4}} \]

Resolving power \( \mathcal{R} \equiv \frac{\lambda}{\Delta\lambda_{\text{min}}} \)

Exercise:
Design a tunable filter that can separate \( \lambda_1 = 1550.918 \text{ nm} \) and \( \lambda_2 = 1552.524 \text{ nm} \) (Two standard wavelengths in optical fiber communication systems).
MEMS Tunable Filters and Modulators

Output fiber

Ground

Top curved mirror

Bottom mirror

Via-hole silicon

Input fiber

Filter Tuning plot

Transmission (dB)

Wavelength (nm)

0 8 V

12 V

15 V

18 V

21 V

V drive

3 \lambda_0/4 < gap < \lambda/2

Reflectivity

Wavelength (nm)

0.0

0.1

0.2

0.3

0.4

0.5

0.6

0.7

1300

1400

1500

1600

gap 3\lambda_0/4

\lambda_0

gap \lambda_0/2

Two-Wave Interference — Different $f$ and $\lambda$

$$E_1 = E_0 \cos(k_1 x - \omega_1 t)$$
$$E_2 = E_0 \cos(k_2 x - \omega_2 t)$$
$$E = E_1 + E_2 = 2E_0 \cos(k_p x - \omega_p t) \cos(k_g x - \omega_g t)$$

$$k_p = \frac{k_1 + k_2}{2}, \quad \omega_p = \frac{\omega_1 + \omega_2}{2}$$
$$k_g = \frac{|k_1 - k_2|}{2}, \quad \omega_g = \frac{|\omega_1 - \omega_2|}{2}$$

$\cos(k_p x - \omega_p t)$: Average

$\cos(k_g x - \omega_g t)$: Modulation, beating

Beat frequency $\omega_b = 2\omega_g = |\omega_1 - \omega_2|$
Phase Velocity, Group Velocity, and Dispersion

Assume the two waves are close in $\omega$ and $k$.

**Phase velocity:**

$$v_p = \frac{\omega_p}{k_p} = \frac{\omega_1 + \omega_2}{k_1 + k_2} \approx \frac{\omega}{k} = \frac{c}{n}$$

**Group velocity:**

$$v_g = \frac{\omega_g}{k_g} = \frac{\omega_1 - \omega_2}{k_1 - k_2} \approx \frac{d\omega}{dk}$$

$$v_g = v_p \left[ 1 + \frac{\lambda}{n} \frac{dn}{d\lambda} \right]$$

Dispersion. $n = n(\lambda)$. Light with different wavelength travels with different velocity in a medium.

Normal dispersion: $dn/d\lambda < 0$, $v_g < v_p$.

$v_g$ determines the speed with which energy is transmitted. It is the directly measurable speed of the wave.
Dispersion in an Optical Fiber

Group index:

\[ v_g \equiv \frac{c}{N} \]

\[ N = n - \lambda \frac{dn}{d\lambda} \]

Dispersion coefficient:

\[ D_\lambda = -\frac{\lambda}{c} \frac{d^2 n}{d\lambda^2} \quad (s/m-nm) \]

→ A measure of time delay per wavelength (nm) after certain distance.

Bandwidth limited by dispersion:

\[ \nu_{\text{max}} L = \frac{0.5}{\delta(\tau / L)} \]

If the original pulse width cannot be neglected compared to the broadening,

\[ \tau_f^2 \approx \tau_0^2 + (\delta\tau)^2 \]

\[ \nu_{\text{max}} = 0.5 / \tau_f \]

Figure 5.6-5  Wavelength dependence of optical parameters of fused silica: the refractive index \( n \), the group index \( N = c_o / c \), and the dispersion coefficient \( D_\lambda \). At \( \lambda_0 = 1.312 \) μm, \( n \) has a point of inflection, the group velocity \( \nu \) is maximum, the group index \( N \) is minimum, and the dispersion coefficient \( D_\lambda \) vanishes. At this wavelength the pulse broadening is minimal.
Pulse Broadening in an Optical Fiber

or,
Multi-Wave Interference — Different Frequencies

At a given position \( r \),

\[
U(t) = \sqrt{I_0} \sum_{q=-(M-1)/2}^{(M-1)/2} \exp[-i2\pi(f_0 + q\Delta f)t]
\]

\[
I(t) = |U(t)|^2 = I_0 \sin^2(M\pi t/T_F) \frac{\sin^2(\pi t/T_F)}{\sin^2(\pi t/T_F)}
\]

\[
T_F = \frac{1}{\Delta f}
\]

(Compare this with multi-wave interference of same frequencies but different phases.)

E.g., 1 ps pulse can be generated by combining 1000 waves separated by 1 GHz from each other. (Note: The exact relation between pulse width and bandwidth depends on the pulse shape. This is a subject of Fourier Optics.)